CHAPTER 2

Graphical Mechanics and Intermittent Motion

Several concepts in basic mechanics were reviewed in Chapter 1, as well as methods of handling force and velocity vectors when analyzing mechanism designs. But all this involved only instantaneous velocities or instantaneous forces. Most design situations are concerned with a continuous spectrum of forces and velocities. If we had to compute these things separately for each instant of a long and complex machine cycle using the techniques of Chapter 1, alone, it would soon prove to be too difficult. Fortunately, in most design situations, approximate "motion curves" of force and velocity can be constructed, showing the complete and continuous variation of these parameters, and the need to find exact instantaneous values of force and velocity will exist only for a few special points on the curves (maximums and minimums, for example). Frequently, these approximate curves will be constructed using design information; e.g., when a spring is to be supplied capable of producing a certain force to drive given mechanisms. Some curves will result from experimental information, as described in Chapter 6, where a transducer, fastened to the machine, produces electrical signals that are "plotted" by a strip chart recorder or oscilloscope. And finally, there are curves derived from others, in some situations. In this chapter we will examine these motion curves, and see how they relate to each other.

This will also lead into a discussion of acceleration and displacement. These parameters will be handled as scalar quantities rather than vector quantities (which is why they were not discussed in Chapter 1), since only their magnitudes are of interest here. In conclusion, the annoying, but essential, subject of "units" will be discussed.

Forces on Mechanisms

Every element in a machine or instrument is acted upon by forces produced either by springs; electrical, pneumatic, or hydraulic motors; gravitational or magnetic fields; or by interactions with other machine elements and structures. In general, such forces can be considered to be either driving forces or load forces, and can be expressed as functions of time or functions of displacement. The discussion will be restricted almost exclusively to forces which are described as functions of time, but will also briefly cover the case of force versus displacement.

Forces, as have been shown, are vector quantities and are, therefore, directional in nature. It is useful to adopt "plus- or minus-sign rules" to describe force vectors so that our theoretical models can account for the fact that some forces act together while others act to oppose each other. For the purpose of this text, forces will be considered to be *positive* if they are directed toward the right, or vertically upwards. They will be considered to be *negative* if they are directed towards the left, or vertically downwards. These and other sign and symbol conventions used in this text are illustrated in Appendix 1.

Force and Intermittent Motion

Let us start our study of force with a very simple and tangible case of intermittent motion. In Fig. 2–1, a finger applies a brief push to a small block resting on a table. This produces a drive force F, on the stone and inevitably, a friction reaction force f_r. After the finger has stopped pushing (comes to rest), the block coasts for a while until the frictional force brings it to a rest. This action, plus the resulting graph of net horizontal force (drive force minus friction force) on the block, is shown in Fig. 2–2, A and B.

It would be quite easy to measure the drive force involved here (with a push scale for example). Measuring the friction force would be a slightly more difficult problem, but it can be done by tipping the table to determine the sliding angle, for example, and a graph can be produced of the force as a function of time, such as is shown in Fig. 2–2B.

This would immediately disclose one important fact: the positive and negative areas enclosed by the force curve (and time axis) must be equal to each other if the velocity of the block is to be zero at the beginning and end of the period being studied. In other words, if the block is to be a true "intermittent motion device" it must start with zero velocity and end there. This equality of the positive and negative areas enclosed by a force (or torque) curve is an important and basic concept in the study of intermittent motion.

Notice that in the graph, Fig. 2–2B, force is expressed in pounds, and time in seconds. These are two quantities of the so-called English system of units which will be used throughout this text and which is tabulated in the appendix. Also given in the appendix, however, is a table of alternate sets of units involving ounces, grams, inches, kilograms, etc. Be consistent and stay within one set of units (one column in the table) for a given problem; it is

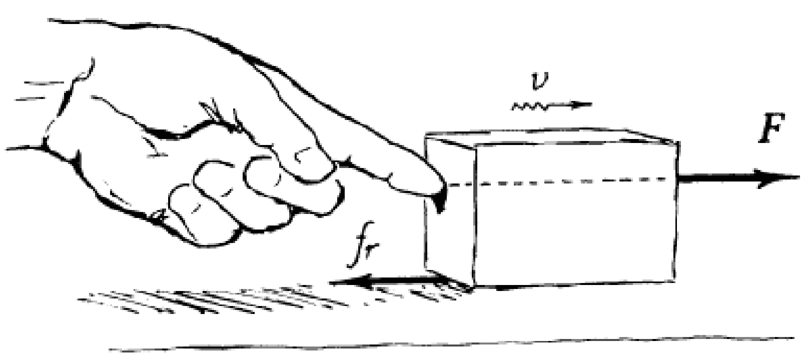

Fig. 2–1. Forces acting on a block being pushed along a flat surface.

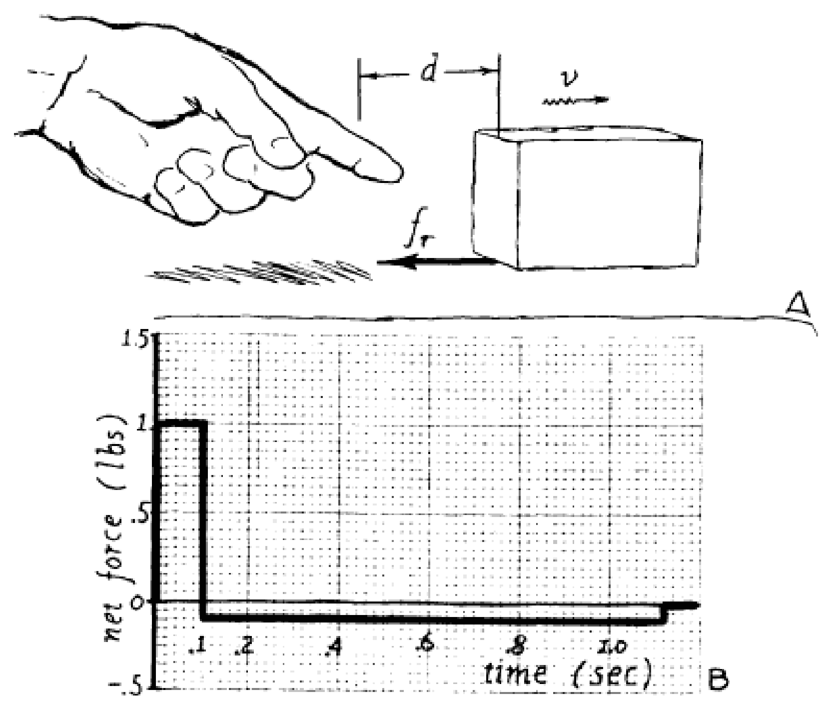

Fig. 2–2. (A) Forces acting on a sliding block; and (B) Graph showing the net or resultant horizontal force on the block as a function of time.

only when the sets of units are intermixed that you will get into difficulty.

Consider now, another topic, the acceleration of the body and its relationship to the forces on that body.

Acceleration

Whenever an object changes its rate of motion; whenever it speeds up or slows down, it experiences positive acceleration (an *increase* in speed), or negative acceleration (a *decrease* in speed). The latter, of course, is sometimes called deceleration but "negative acceleration" is the more accurate term for theoretical model builders. Acceleration is defined as the rate-of-change of velocity as a function of time. It is a very important parameter in machine design and is, therefore, worthy of our attention. Fortunately, once the forces on a body are known, its acceleration can readily be determined. The acceleration (as a function of time) is found by using Newton's famous Second Law:

$$F = ma$$

force = mass × acceleration
(units: lbs = slugs × ft/sec/sec = slugs × ft/sec^2)

This law tells us that the acceleration of a body at any instant is equal to the force on the body at that instant divided by the inertial mass of the body. A force curve, then, can be converted to an acceleration curve merely by dividing each point on the force

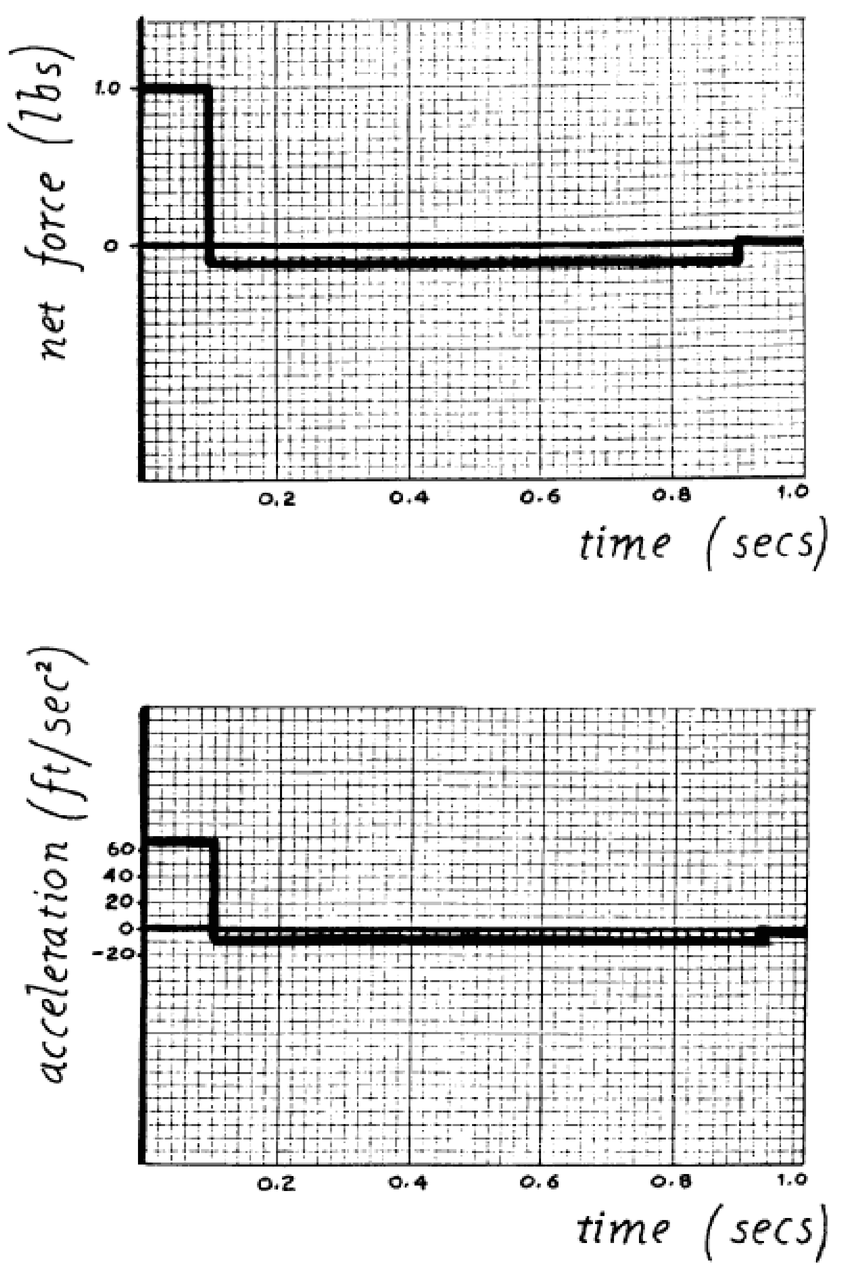

Fig. 2-3. Relationship between acceleration and force curves for the block-on-the-table problem.

curve by the mass of the body being studied. This can be done by drawing a new acceleration curve using information obtained from the force curve; or by merely changing the vertical scale on the original force curve. Figure 2-3 shows the relationship between the forces and accelerations on the block. Note that the positive and negative areas enclosed by the acceleration curve are equal for intermittent motion.

The units for acceleration are feet per second per second, which equals feet per second, squared; again, in the English system of units. Time is still given in seconds. Mass has been expressed in slugs. I do not know the origin of the name "slugs" nor why the word was adopted. By definition (and using Newton's equation of $F = ma$), a unit force of 1 pound will produce an acceleration of 1 foot per second, squared, in a 1-slug mass: thus a slug is related to more familiar units by the following equation:

$$M = f/a$$

$$\text{One slug} = \text{one}\,\frac{\text{lb-sec}^2}{\text{ft}}$$

—if that is any help! We can find the mass of a body by dividing its sea-level weight, in pounds, by 32. (32 ft/sec^2 being the acceleration of gravity in English units. 32.16 or 32.2 are sometimes used but 32 is close enough for our purposes.)

Once an acceleration curve is achieved, a velocity curve can be constructed. Velocity is also of interest in design work, and is the next topic to be discussed.

Velocity

Velocity is defined as the rate-of-change of position of a body with time. It is a measure of the speed at which a body is moving; and the direction in which it is moving. Because velocity, like force, is a vector, positive and negative sign conventions must be adopted. In this text we will assume that velocity to the right, or upward, is positive: and velocity to the left, or downward, is negative.

If the acceleration of a body as a function of time is known, a velocity curve can be constructed by measuring the area enclosed by the acceleration curve and the horizontal (time) axis on the acceleration diagram. This area could be found by a mathematical process called *integration* if the acceleration curve could be described mathematically. There are many special cases in which this is possible, and some students struggle through a course in integral calculus to learn them. With a drawing of an acceleration curve, however, the integration process can be performed graphically, merely by counting the squares enclosed by the curve and the time axis, by using an instrument called the planimeter, or by a series of algebraic computations in which the area under a curve is approximated by a series of rectangles, triangles, trapezoids, and other simple forms whose areas are easy to compute. The same techniques can be used for finding the areas under complex curves as for finding those areas under simple curves. It is never necessary to learn special procedures for special cases; an advantage of the graphical over the mathematical approach.

The integration process is started at the "velocity equals 0" point. This is not an arbitrary choice, it is necessary to start at the zero velocity point to avoid what mathematicians call the "constant of

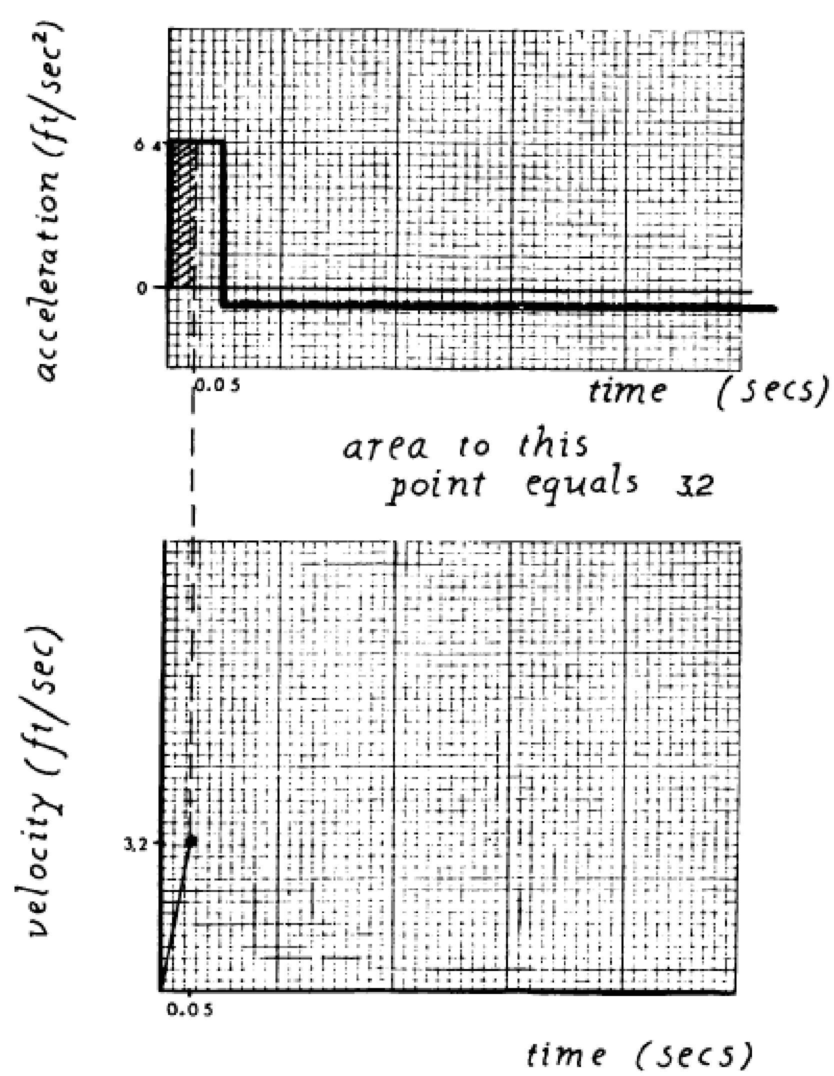

Fig. 2-4. Deriving a velocity curve from an acceleration curve—first step.

integration." The velocity of the body after 0.05 second of acceleration is equal to the total area enclosed under the acceleration curve from the time (or velocity) equals zero point to the time equals 0.05 second point. The velocity of the body after 0.1 second of acceleration is equal to the total area under the acceleration curve from time = 0, to time = 0.1 second, etc., as shown in Figs. 2-4 and 2-5.

When the acceleration curve passes below the time axis it produces the "negative area" which must be subtracted from the total area accumulated to that point, as shown in Fig. 2-6. In an intermittent motion system, the velocity curve starts and stops at zero; thus, as shown in Fig. 2-7, as much area is eventually subtracted as was added. This process of graphical integration is very important as the methods of this text cannot be used unless the process is thoroughly understood. Re-read the above and study the illustrations until the process of converting the "force-on-the-block" curve into a velocity curve is clearly understood.

The integration process can also be used to shed light on the "units" that should be used for the function being derived. Acceleration, as was seen, has the units of feet per second, squared. Time is measured in seconds. The area enclosed by an acceleration curve and the time axis, therefore, would have the dimensions of:

$$\text{Area} = \text{height} \times \text{base}$$

$$\text{Units: area} = \frac{\text{ft}}{\text{sec}^2} \times \text{sec}$$

It is a very useful, but little-known fact that "units" can be treated as algebraic quantities in trying to simplify an expression or to see where an equation is heading. For example, just as:

$$\frac{A}{B^2} \times B = \frac{A}{B}$$

so does the original "units" expression given above,

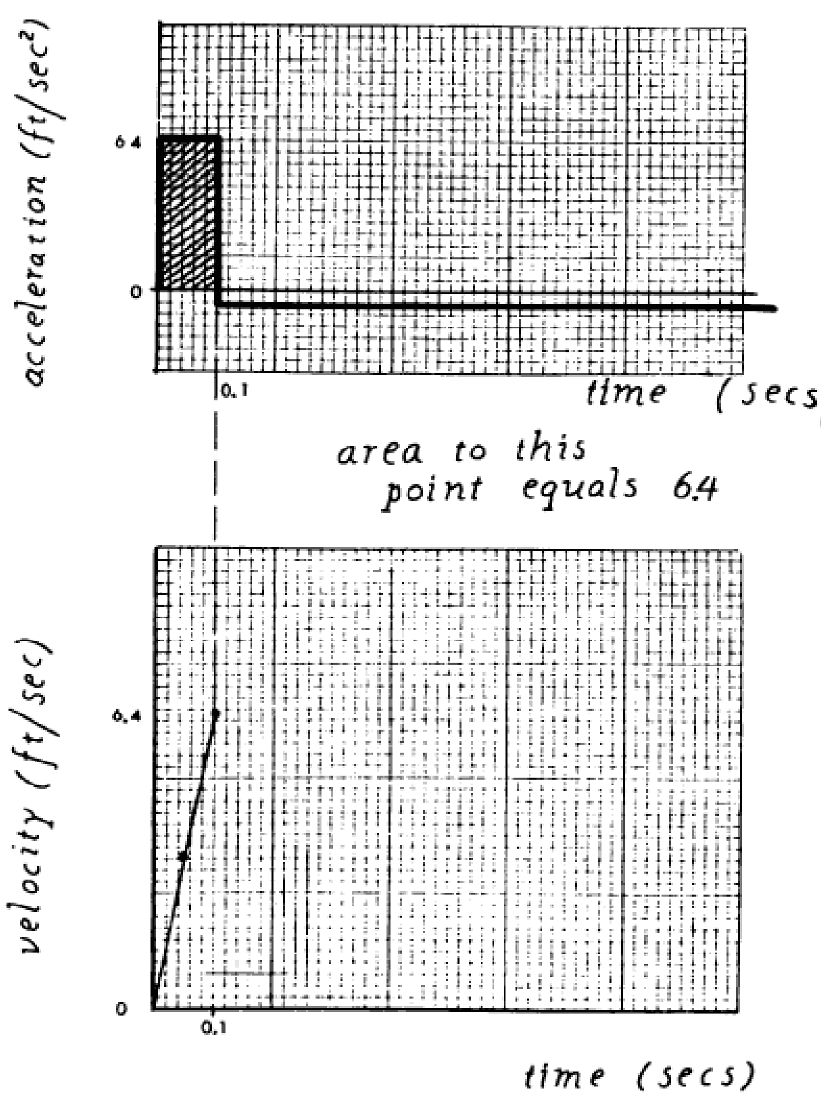

Fig. 2-5. Deriving a velocity curve from an acceleration curve—second step.