CHAPTER III.

PLANE MECHANISMS CONTAINING ONLY TURNING PAIRS.

26. Quadric Crank-chains.—If we endeavor to make a plane mechanism out of links containing only turning pairs, we find that the least number of links with which this can be done is four. A chain of *three* links so connected forms

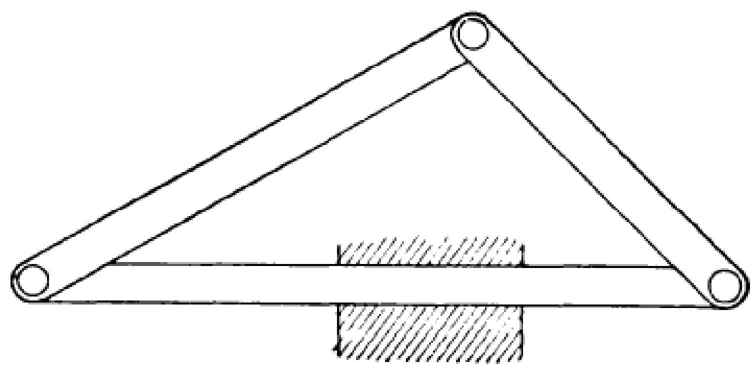

FIG. 39.

an arrangement which is of value as a *structure* (a simple triangular roof-truss), but is of no service as a mechanism, since its parts can have no relative motion.

On the other hand, a simple chain of *five* or any greater number of links connected by turning pairs is equally useless as a mechanism, since the relative motion of at least two of its links is not constrained, as has been shown in § 3.

Let us consider, therefore, a chain of four links connected by turning pairs whose axes are parallel. When the links of this chain are of unequal lengths the smallest is called the crank, and since the four links form a quadrilateral, the chain has been called by Reuleaux * the *quadric (cylindric) crank-chain*. The term 'cylindric' distinguishes this chain

* Kinematics of Mach., §§ 62–65.

from the corresponding spheric chain, in which the axes are not parallel.

In quadric crank-chains it will be convenient to distinguish between links having a swinging or partial turning movement and those which can execute complete rotations relatively to the fixed link in the chain.

The former links will be called levers, the latter cranks. It is obvious that by altering the relative lengths of the links we can obtain different relative motions, and hence different mechanisms. From these, again, other different mechanisms are produced by inversion of the chain.

27. Virtual Centres and Centrodes.—Let *abcd*, Fig. 40, represent the four links of a quadric crank-chain. Each of these links will have motion relatively to every other, and hence we shall have six virtual centres. Four of these centres are readily identified as the axes of the turning pairs; for instance, the virtual centre of *c* with regard to *d*, or of *d* with regard to *c*, is obviously the point 3, and may be

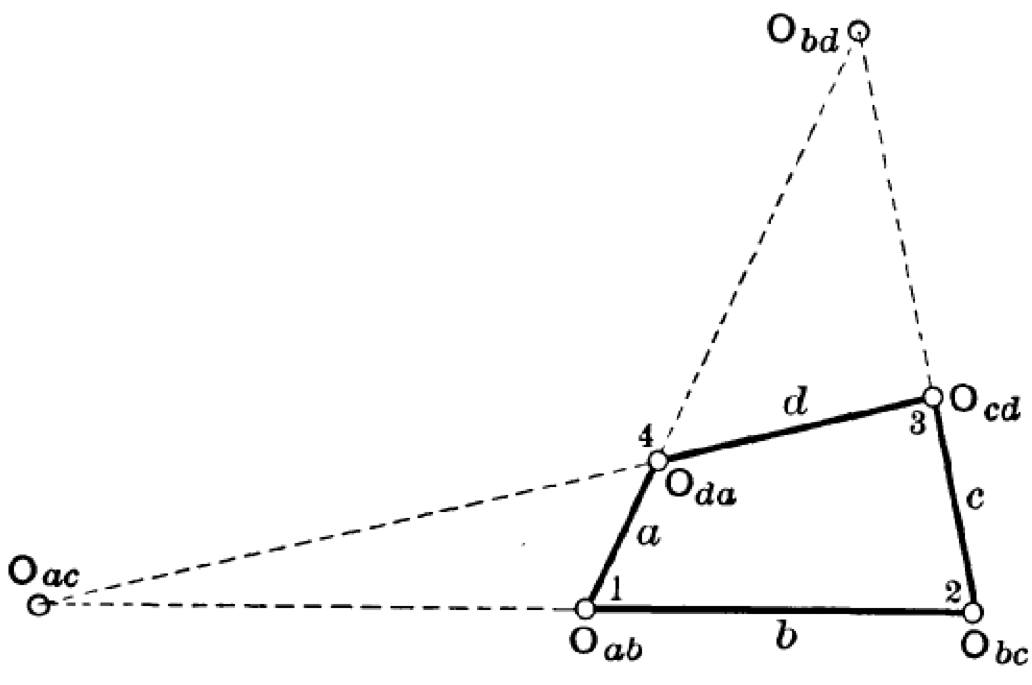

FIG. 40.

indicated as O_{cd} or O_{dc}. In the same way we have O_{ad}, O_{ab}, and O_{ac}; all these points are in fact permanent centres as regards their own pair of links. Remembering that for any three bodies having plane motion the three virtual centres lie in one straight line, it is easy to see that O_{ac} must lie at the join of the straight lines drawn through O_{ab} and O_{bc}, and through O_{ad} and O_{cd}. In the same way O_{bd} is at the intersection of the lines $O_{ab}O_{ad}$ and $O_{cb}O_{cd}$.

Supposing b to form the *frame* or fixed link, it is seen that since O_{ab} and O_{bc} are permanent centres, the centrodes of a and c with regard to b are points, namely O_{ab} and O_{bc}.

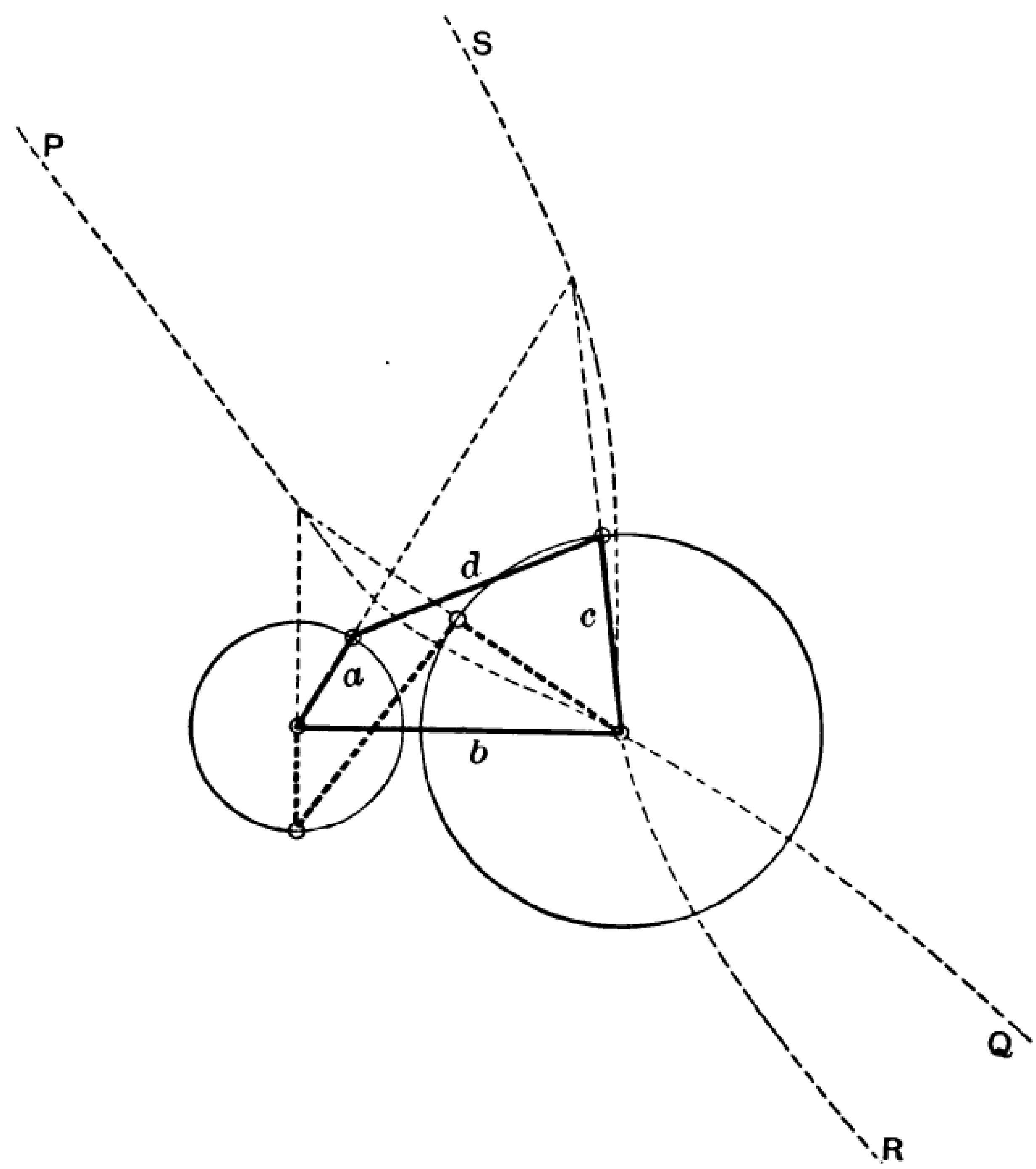

FIG. 41.

The centrode of d with regard to b is the locus of O_{db} and takes the form of a curve having four infinitely distant points; portions of it are readily drawn by finding a series of positions of O_{db} corresponding to successive positions taken up by the three links a, c, and d. In a similar way may be obtained the centrode of b with regard to d (supposing d to be the fixed link). The curve $PQRS$

in Fig. 41 represents the centrode of *d* with regard to *b*; the construction for two points on the curve is shown.

28. Angular Velocities.—It is frequently of importance, having given the angular velocity, say, of the link *a*, to find

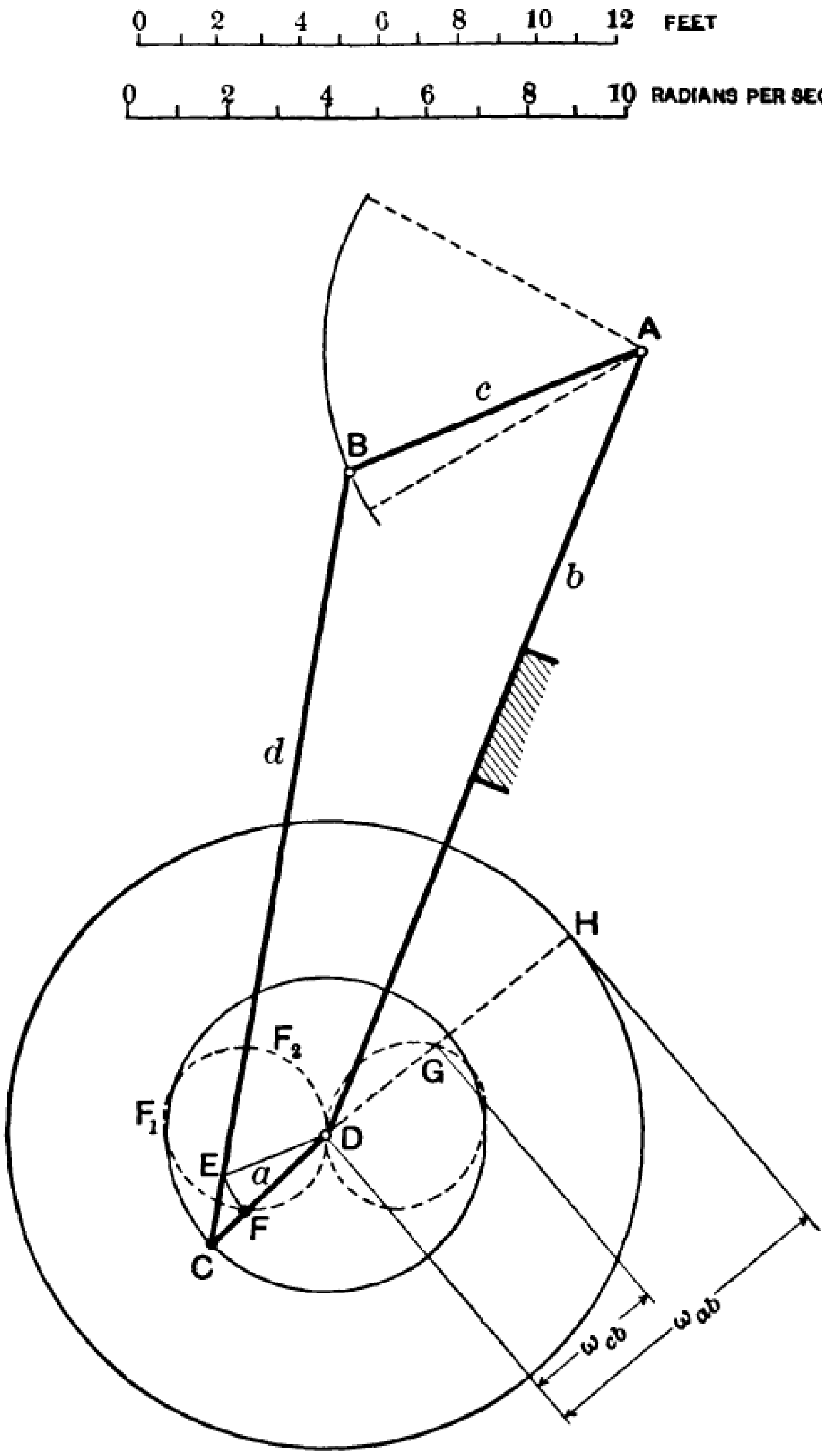

FIG. 42.

that of any other link, say *c*, or, in other words, to determine the angular velocity ratio of the chain. This can be very simply done by construction.

In Fig. 42 let *ABCD* represent the mechanism, *AD* being the fixed link, and the uniform angular velocity of CD being known. It is required to determine the angular velocity of AB for any position of the mechanism.

Draw DE parallel to AB, and cutting BC, or BC produced, in E. With centre D and radius DE mark off DF

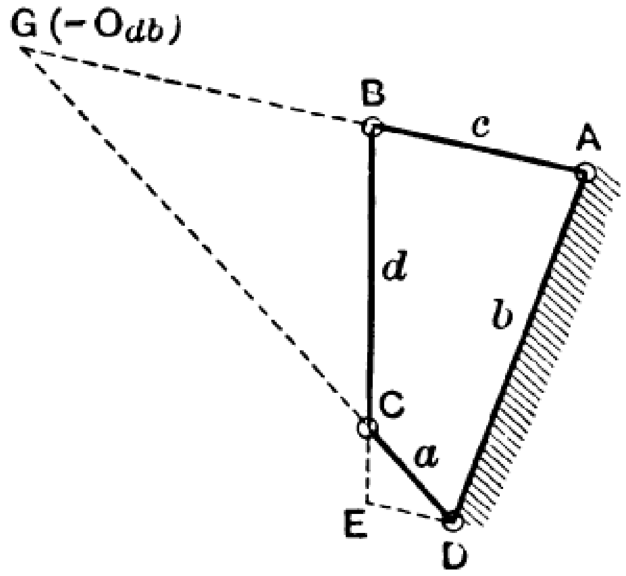

FIG. 43.

along DC. Then DF represents the angular velocity of c on the same scale as that on which AB represents the angular velocity of a, and if a series of points such as F be obtained, the curve FF_1F_2G . . . drawn through them will form a polar diagram of angular velocities for c and a.

To prove this construction, let ω_{cb}, ω_{ab} be the angular velocities of c and a respectively with regard to b. In Fig. 43 find O_{db}, the intersection of AB and DC at G, and draw DE parallel to AB, meeting BC in E.

Since the link d is turning for the instant about G, we must have

$$\frac{\text{linear velocity of } B}{\text{linear velocity of } C}=\frac{GB}{GC}.$$

Now $$\omega_{cb}=\frac{\text{linear velocity of } B}{AB},$$

and $$\omega_{ab}=\frac{\text{linear velocity of } C}{CD};$$

hence $$\frac{\omega_{cb}}{\omega_{ab}}=\frac{\text{linear velocity of } B}{\text{linear velocity of } C}\cdot\frac{CD}{AB}$$

$$=\frac{CD.GB}{AB.GC}.$$

But by construction the triangle BGC is similar to the triangle EDC; hence

$$\frac{GB}{GC}=\frac{ED}{DC}$$

Therefore $$\frac{\omega_{cb}}{\omega_{ab}}=\frac{CD.ED}{AB.DC}=\frac{ED}{AB}$$

Thus if AB represents the angular velocity of a with regard to b, ED represents on the same scale that of c with regard to b.

Fig. 42 gives such a velocity diagram, drawn to scale, for the beam of a beam-engine when the crank rotates uniformly. For comparison the circle of radius $DH=AB$ has been drawn, so that for any radius DGH the intercept DG represents ω_{cb}, just as DH represents ω_{ab}. The polar curve of velocity is shown by a dotted line.

The distances taken are:

$$AB=8 \text{ feet}=DH;$$
$$BC=20 \text{ feet};$$
$$CD=4 \text{ feet};$$
$$DA=21.5 \text{ feet}.$$

When the crank is in the position DH the angular velocity ratio is

$$\frac{DG}{DH}=\frac{3.5}{8}=0.438,$$

or at that particular instant the beam is swinging with 0.438 the angular velocity of the crank. If the crank rotates uniformly at 60 revolutions per minute or 6.28 radians per second, in the position AB the beam is moving with an angular velocity of $6.28 \times 0.438 = 2.75$ radians per second.

From the curve of angular velocity thus obtained we might draw the curve of angular acceleration by the construction described in § 22. Notice that the construction just described can still be applied in positions of the mechanism where O_{bd} is inaccessible, i.e., when AB and CD are nearly parallel, and when the relative angular velocities, therefore, could not be found from the position of the virtual centres.

29. Inversions of the Quadric Crank-chain.—In the particular example of the quadric crank-chain just examined, the lengths of the links are such that while the link a executes complete rotations with reference to b or d, c only swings. a is then a crank, c a lever, and if the link b is the fixed one, the resulting mechanism is called the lever crank-chain.

In order that a may execute complete rotations with regard to b it is necessary that $a+b \leqq c+d$, while also $a+d \leqq c+b$, a being the smallest of the links.

With these proportions let us see the result of inversion of the chain. On considering the relative motions of the links we find that the motion of a relatively to b or d is that of complete rotation, while with regard to either of the

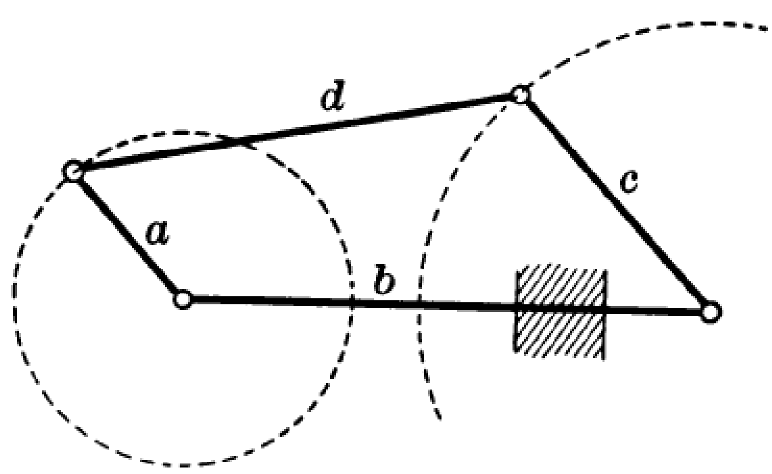

FIG. 44.

same links c only swings or performs partial revolutions. As has been already pointed out, inversion can make no