CHAPTER IV.

SLIDER-CRANK CHAINS.

34. Slider-crank Chain.—A very important chain is obtained from the quadric crank-chain by substituting a sliding pair for one of the turning pairs. It is obvious that the links will undergo the same relative change of position in Fig. 60 (*b*) as in Fig. 60 (*a*), although the lever *c* has been replaced by a block sliding in a circularly curved slot of the same radius as the original lever. The chain as thus transformed may be called a cylindric slider-crank chain, although this name is generally applied to the particular case in which O_{cd} is at an infinite distance and the block slides in a straight slot. It is plain that the mechanism of Fig. 60 (*c*) may be obtained from that of Fig. 60 (*b*) by continually increasing the radius of the pair *cd* until it becomes infinite. The pair *cd* may have prismatic surfaces of any form so long as the sliding motion is properly constrained; thus, for example, *c* may be a hollow block sliding on a prismatic rod *d*, Fig. 60 (*c*). The slider-crank chain in its cylindric form has of course plane motion, and is of special importance, since its different inversions form amongst others the mechanisms of various types of reciprocating steam-engines.

The six virtual centres of the slider-crank chain are easily found, exactly as in the case of the quadric crank-chain, but O_{cd} is always inaccessible. Fig. 61 shows the centrodes of the links *b* (representing the connecting-rod of a direct-acting engine) and *d* (representing the frame or bedplate). The centrode of *b* with respect to *d* (i.e., if *d* is considered as the fixed link) is shown by the full line; the dotted curve represents the centrode described by O_{bd} if b

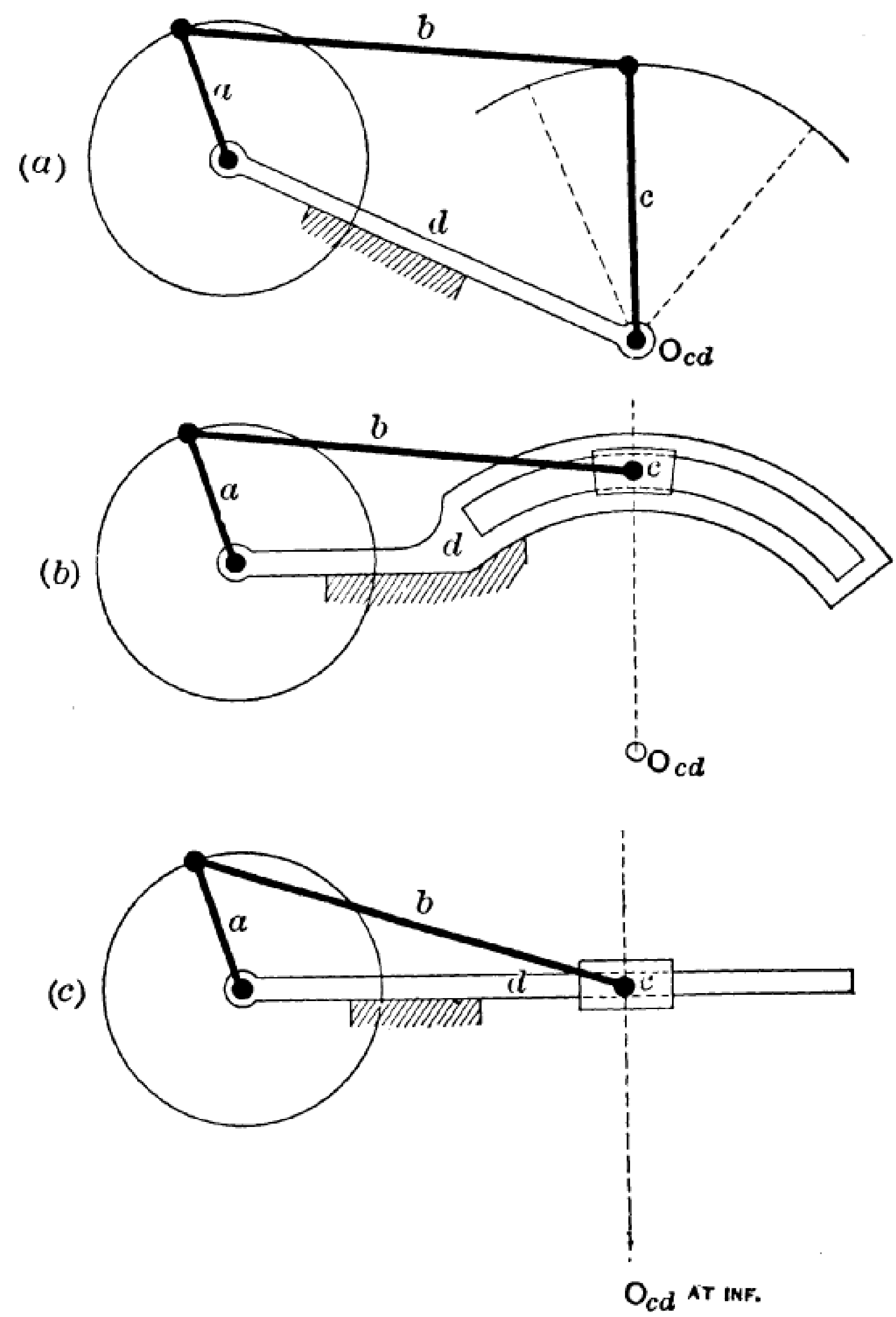

FIG. 60.

is taken as the fixed link. The construction for one point is shown in each case.

When d is fixed the link c represents the piston, piston-rod, and cross-head of the same machine. The link a represents the crank, and b the connecting-rod. A point on the link b between A and B describes an oval curve with reference to d, the shape depending on the position of the point selected, and on the ratio of the lengths of crank and con-

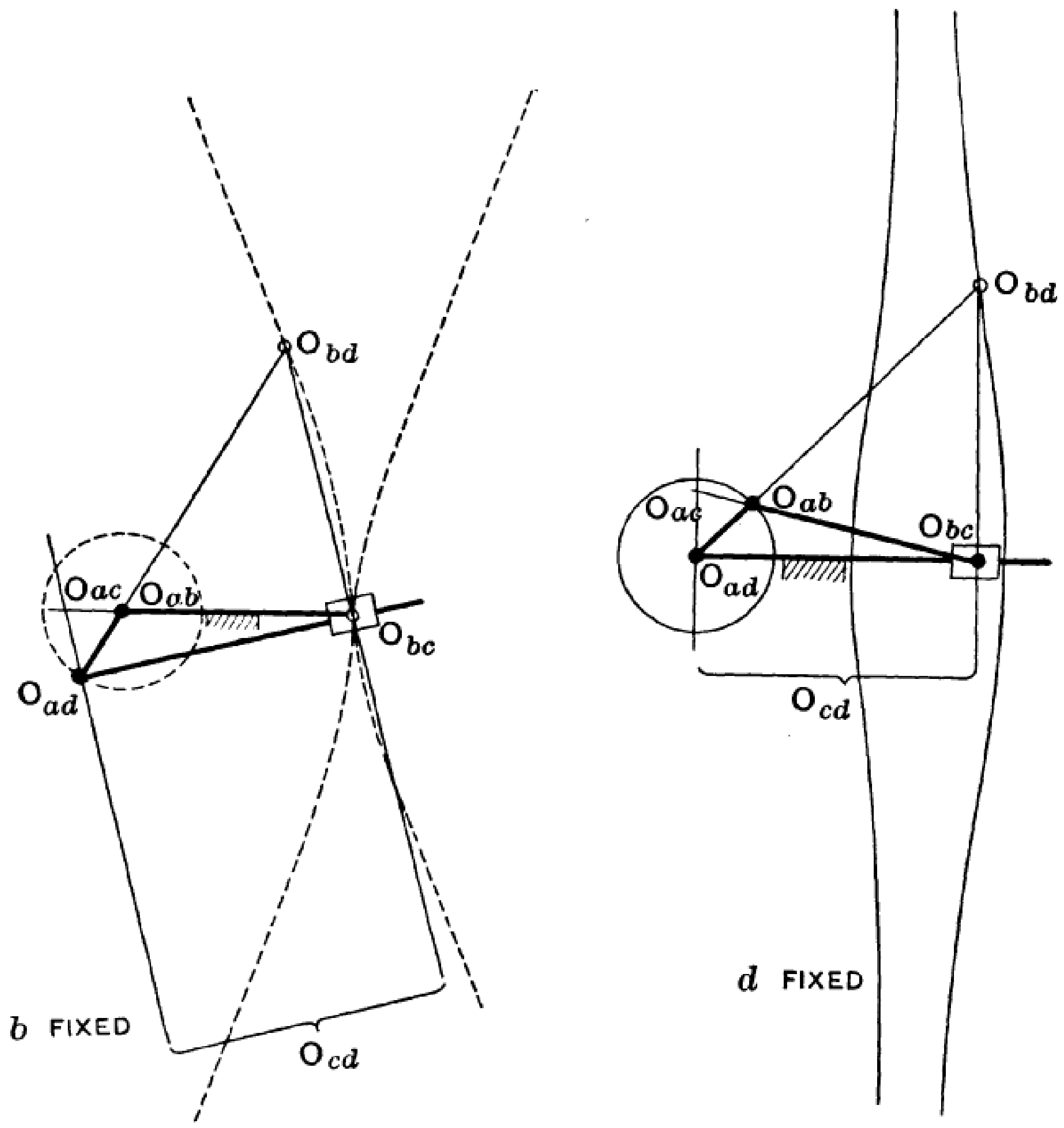

FIG. 61.

necting-rod. This fact is utilized in the design of certain valve-gears.

35. Displacement, Velocity, and Acceleration of Cross-head in Direct-acting Engine. (**First Inversion of Slider-crank Chain.**)—One of the most important problems in connection with the slider-crank chain is the determination of the velocity and acceleration of the link c, Fig. 60, supposing d to be fixed, and a to rotate with uniform angular velocity. This is approximately the case in a direct-acting steam-engine, where c would represent the cross-head and b the connecting-rod.

It is in general most convenient to deal with these problems graphically, but we shall first give an analytical investigation.

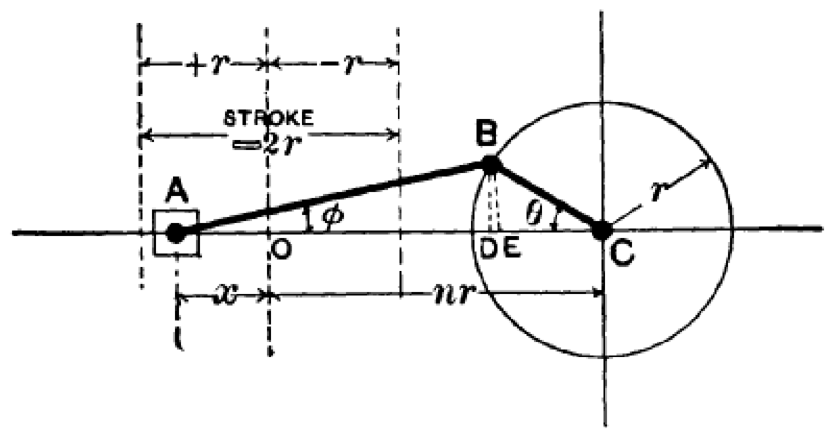

FIG. 62.

In Fig. 62, suppose the line of stroke AO to pass through C, the centre of the crank-shaft. Let BC (the throw of the crank) $=r$, and let $\frac{AB}{BC}=n$, so that the length of connecting-rod $=nr=AB$. When the crank makes any angle θ with the centre line AC, let x be the distance of the cross-head A from O, the middle of its stroke. Draw BD perpendicular to AC, and mark off $AE=AB$. If φ is the angle of obliquity of the connecting-rod,

$$\sin\varphi=\frac{\sin\theta}{n}, \quad \text{and}$$

$$\cos\varphi=\frac{1}{n}\sqrt{n^2-\sin^2\theta}.$$

Now
$$\begin{aligned} x&=AC-OC=AC-AB \\ &=CD+DA-AB \\ &=r\cos\theta+nr\cos\varphi-nr \\ &=r\left(\cos\theta-n+\sqrt{n^2-\sin^2\theta}\right). \quad . \quad . \quad . \quad . \quad (1) \end{aligned}$$

This gives x in terms of the crank angle θ. It is to be noticed that when $\theta=\frac{\pi}{2}$ the cross-head is not at the middle of its stroke, but at a distance

$$\begin{aligned} x_0&=r(\sqrt{n^2-1}-n) \\ &=-\frac{r}{\sqrt{n^2-1}+n}, \end{aligned}$$

the negative sign indicating that A is now to the right of O, Fig. 62.

In the case of a cross-head having simple harmonic motion we should have simply

$$x = r\cos\theta.$$

The term $r(\sqrt{n^2-\sin^2\theta}-n)$ in equation (1) thus gives what is called the "error due to obliquity" of the connecting-rod. Its values for $\theta=\frac{\pi}{2}$ are shown below for some usual values of n.

$n=$	4	5	6
$\sqrt{n^2-\sin^2\theta}-n=$	-0.13	-0.11	-0.09

The error due to obliquity is thus seen to diminish rapidly as n increases.*

Next, to determine the velocity of the piston at any instant we differentiate x with regard to time and obtain

$$\frac{dx}{dt}=r\left[-\sin\theta\frac{d\theta}{dt}+\frac{1}{2}(n^2-\sin^2\theta)^{-\frac{1}{2}}\frac{d}{dt}(n^2-\sin^2\theta)\right]$$

$$=-r\frac{d\theta}{dt}\left[\sin\theta+\frac{2\sin\theta\cos\theta}{2\sqrt{n^2-\sin^2\theta}}\right].$$

This is not very convenient for use in practice, but for ordinary values of n we may write without large error n instead of $\sqrt{n^2-\sin^2\theta}$. For example, if $\theta=\frac{\pi}{2}$, and $\sin\theta$ has its greatest value,

$\sqrt{n^2-\sin^2\theta}=$	3.87	4.89	5.91
when $n=$	4	5	6

Further, we may write V_c, the linear velocity of the crank-

* For a discussion of the problem of the connecting-rod see Hill, Min. Proc. Inst. C. E., Vol. CXXIV, p. 390. Also consult Unwin, Min. Proc. Inst. C. E., CXXV, p. 363, and a paper by G. A. Burls, Min. Proc. Inst. C. E., Vol. CXXXI, p. 338.

pin, instead of $r\frac{d\theta}{dt}$, and, omitting the negative sign, which simply shows that x diminishes at first while θ increases, we have very approximately for the velocity of the piston or cross-head

$$V_p = V_c\left(\sin\,\theta + \frac{\sin\,2\theta}{2n}\right). \quad . \quad . \quad . \quad . \quad (2)$$

As an example, suppose an engine 12 inches stroke running at 250 revolutions per minute, the length of connecting-rod being 3 feet. The crank-pin velocity will be $\frac{250 \times 3.14}{60} =$ 13.08 feet per second. When $\theta = 45°$, the value of n being 6, we have, from equation (2),

$$\begin{aligned} V_p &= 13.08(0.70711 + 0.08333) \\ &= 13.08 \times 0.79044 \\ &= 10.340 \text{ feet per second.} \end{aligned}$$

If the velocity were calculated from the accurate expression previously obtained, we should get

$$\begin{aligned} V_p &= 13.08\left(0.70711 + \frac{1}{2\sqrt{36 - 0.4998}}\right) \\ &= 13.08 \times 0.79103 \\ &= 10.348 \text{ feet per second.} \end{aligned}$$

The approximation, therefore, has led to an error of only 0.008 foot per second in this case.

Proceeding to determine the acceleration of the piston or cross-head for any crank angle, we find very approximately from equation (2), remembering that V_c is constant,

$$\frac{d}{dt}(V_p) = V_c\left(\cos\,\theta\,\frac{d\theta}{dt} + \frac{1}{2n}\left[2\,\cos\,2\theta\frac{d\theta}{dt}\right]\right)$$

Now $\frac{d\theta}{dt} = \frac{V_c}{r}$; thus

$$\text{acceleration of piston or cross-head} = \frac{V_c^2}{r}\left(\cos\,\theta + \frac{\cos\,2\theta}{n}\right). \quad (3)$$

The following table gives the value of $\cos\theta + \cos\frac{2\theta}{n}$ for different values of θ and n.

θ.	Value of n.					
	4	4.5	5	5.5	6	∞
0° or 360°.......	1.250	1.222	1.200	1.182	1.167	1.000
30° or 330°.......	0.991	0.977	0.966	0.957	0.949	0.866
60° or 300°.......	0.375	0.389	0.400	0.409	0.417	0.500
90° or 270°.......	−0.250	−0.222	−0.200	−0.182	−0.167	0.000
120° or 240°.......	−0.625	−0.611	−0.600	−0.591	−0.583	−0.500
150° or 210°.......	−0.741	−0.755	−0.766	−0.775	−0.783	−0.866
180°.......	−0.750	−0.778	−0.800	−0.818	−0.833	−1.000
	Values of $\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$					

36. Graphic Methods for Cross-head Velocity and Acceleration.—We proceed to consider graphic means of determining velocity and acceleration for the cross-head or piston of a direct-acting engine. It is of course possible to draw first a curve of displacement on a time base, and then use the methods described in Chapter II, but simpler means can be employed in this case. In Fig. 63 let AB, BC repre-

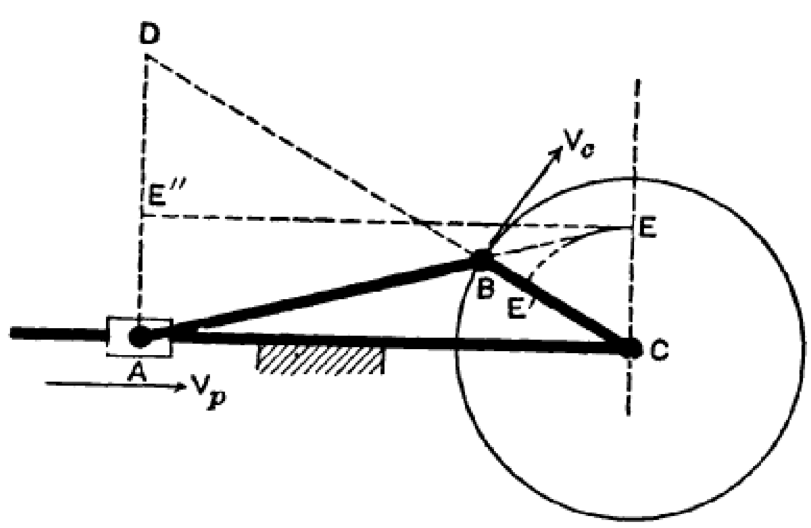

FIG. 63.

sent the connecting-rod and crank in any given position. The point A is moving along the straight line AC, while B is moving for the instant in a direction perpendicular to BC. Hence D, the virtual centre of AB with regard to the fixed