CHAPTER V.

DETERMINATION OF VELOCITY AND ACCELERATION IN PLANE MECHANISMS.

48. Velocity and Acceleration Determined from Virtual Centres.—It is often necessary to determine the magnitude and direction of the velocity or acceleration of a given point of a given link in a plane mechanism. Such a calculation, for example, is frequently required if we wish to find the forces acting on a part of a machine when in motion, with a view to the correct proportioning of such a part to the work it has to do.

We have already studied this problem in certain cases, especially as regards the cross-head of a direct-acting steam-engine; the question has now to be discussed in a more general manner.

In a given mechanism, having given the velocity of a point on one link, and having found the positions of the various virtual centres, it is possible to determine the velocity of any point on any one of the links.

Take for example the beam-engine of Fig. 96, in which we suppose V_c, the velocity of the crank-pin, to be known. It is required to find the actual linear velocity (i.e., the velocity with relation to the frame or fixed link) of the piston and rod *b*.

Let *a* be the fixed link, *b* the piston, *d* the beam, *e* the connecting-rod, and *f* the crank.

First find O_{bd} at the intersection of a horizontal line through the beam centre O_{da} and the line joining O_{bc} and O_{cd}. Note that O_{ba} is at an infinite distance. Next find

O_{df}, draw a horizontal line through O_{af}, and find its intersection with the line joining O_{db} and O_{df}. This point is O_{bf}.

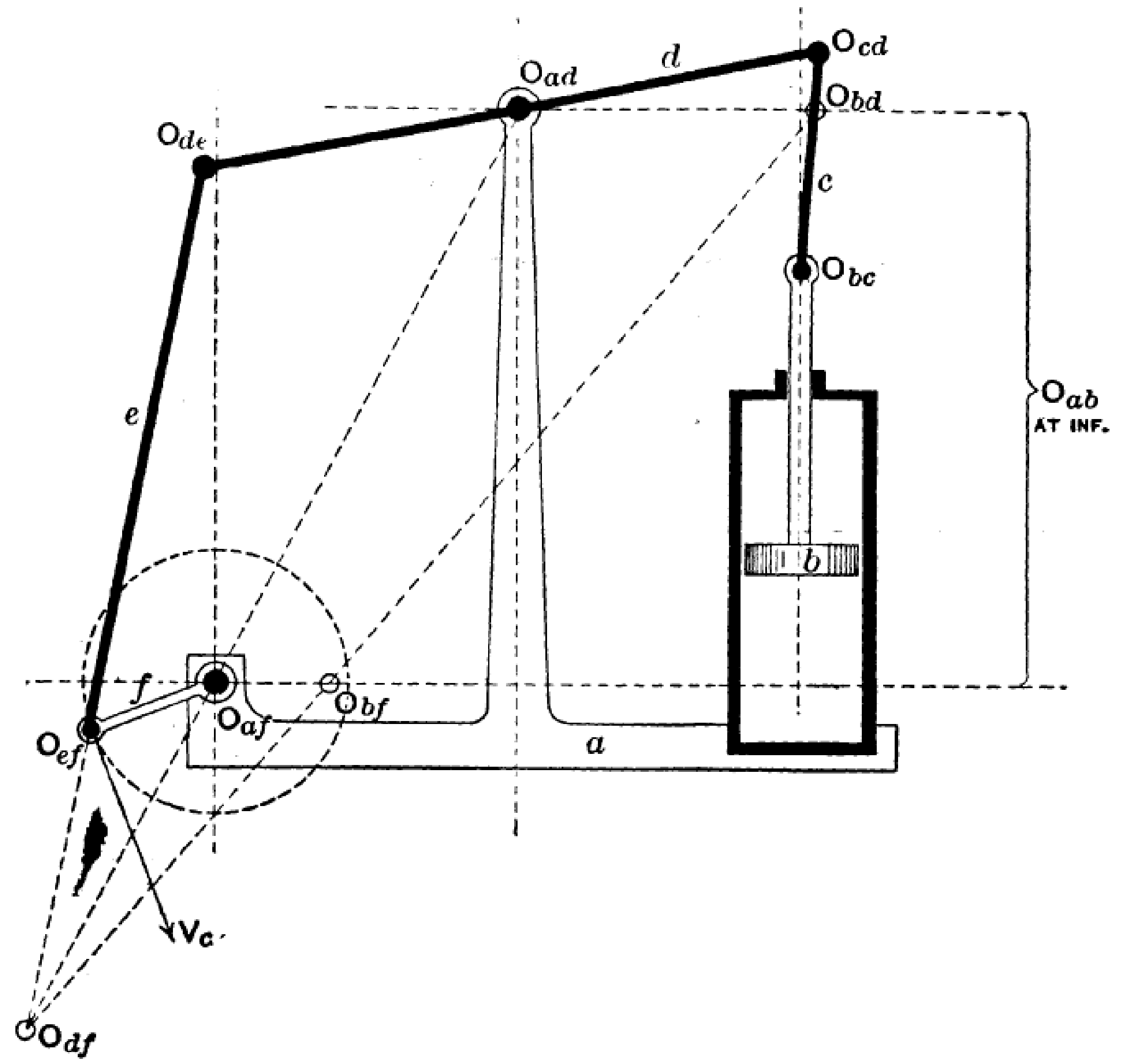

FIG. 96.

All these centres are readily found, remembering that they lie in threes in straight lines.

The point O_{bf} is a point common for the instant to the links b and f. Let the length of crank $= r$ and let the distance $O_{af} \ldots O_{bf} = m$.

Then the actual linear velocity of the point O_{bf} (considered as a point on the link f) must be $V_c \times \frac{m}{r}$, in a direction perpendicular to the line $O_{af}O_{bf}$, and this must also be the velocity of the link b, since O_{bf} is for the instant a point on that link also. The same construction will give the velocity of the piston for any position of the mechanism, except when the crank is on the dead-centre.

A similar method may be used in any case in which the various virtual centres can be found, but is not always possible for all positions of the mechanism, because many of the centres periodically recede to an infinite distance. This fact considerably reduces its practical usefulness.

49. Method by Using Point-paths.—The velocity of a given point on any link may be most simply determined for any given position of a mechanism by carefully drawing (to as large a scale as possible) the mechanism in two positions, one slightly before and the other slightly after the given position. The velocity of some one point of the mechanism being known, the velocity of the given point is readily found by comparing the displacements of the two points in the short time supposed to elapse between the two positions drawn, the direction of motion being known from the point-path on the drawing. It should be noted that this method of finding the velocity required is not susceptible of great accuracy, because the displacements whose ratio is measured must be supposed very small, in order that the result obtained may be as nearly as possible the true velocity of the point when the mechanism is actually in the given position. Hence the ratio of the displacements is difficult to measure. The method is nevertheless often used in practice.

As an example the mechanism of Fig. 97 may be taken. The figure shows Bremme's valve-gear.* It consists essentially of a lever-crank chain, the motion of the valve being taken from a point on one link produced. The figure, necessarily drawn here to a small scale, shows the proportions of an actual gear for a small marine engine. The eccentric of the engine corresponds to the crank of the lever-crank chain, and in practice coincides in angular position on the shaft with the engine-crank. The dimensions are:

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* See *Mechanical World*, September 2, 1889.

$AC = 1\frac{3}{4}''$ (throw of eccentric);
$CE = 10\frac{1}{2}''$;
$CD = 15\frac{3}{4}''$;
$EB = 14''$;
$AB = 20''$ (when the engine is going ahead).

The engine is reversed by altering the position of the suspension point B, as shown by the dotted arc.

It is required to determine the vertical component of the velocity of the point D (from which the valve is driven by a long rod) for any position of the gear, supposing the eccentric AC rotates uniformly at a speed of 170 revolutions per minute.

We first take a number of positions of AC (in this case 12 in one revolution), corresponding to equal small intervals of time (in this case 0.0294 second), and the corresponding positions of E and D are found. They are shown on the diagram and numbered successively, those of D forming points on a closed curve roughly oval in shape.

The vertical displacements of the point D have been plotted on a time base, giving the diagram xx. From this the velocity curve YYY has been drawn by the method of § 18. On determining the scale of the diagram we see that between the positions 4 and 5 the valve moved 0.60 inch upwards in $\frac{1}{12}$ revolution, i.e. in 0.0294 second. The velocity of the valve at the middle of that interval will therefore be approximately

$$\frac{0.60}{12 \times 0.0294} = 1.70 \text{ foot per second.}$$

In the same way the maximum downward velocity of the valve is found to be while the crank is moving from 10 to 11, and its value is about 2.00 feet per second.

If necessary the vertical *acceleration* of the valve can be determined as in § 19.

On drawing out the example for himself the reader will find that even if the mechanism be drawn full size great care is necessary to obtain anything like an accurate result.

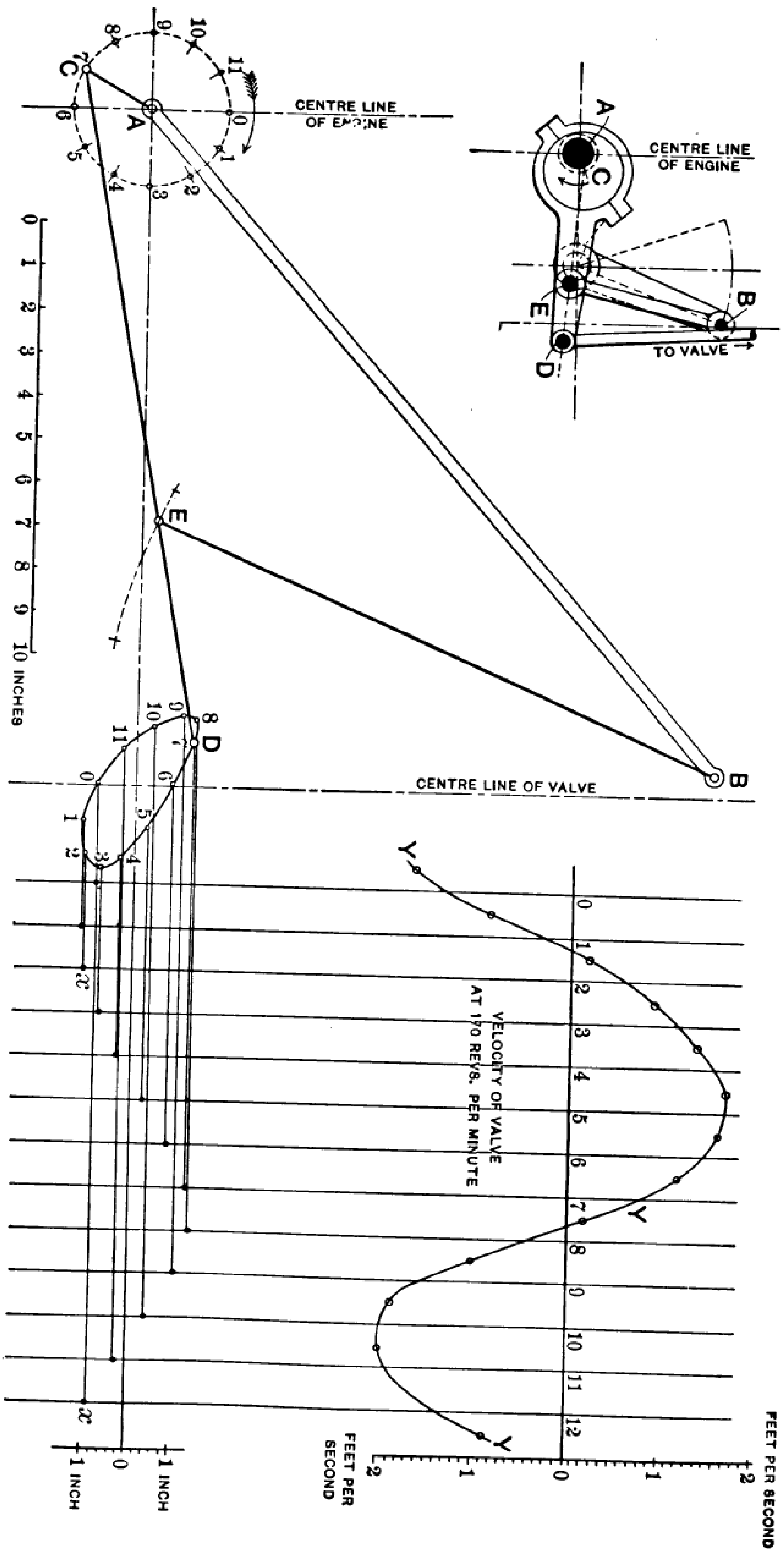
CENTRE LINE
OF ENGINE
CENTRE LINE
OF ENGINE
TO VALVE
CENTRE LINE OF VALVE
10 INCHES
VELOCITY OF VALVE
AT 170 REVS. PER MINUTE
FEET PER SECOND
FEET PER
SECOND
1 INCH
1 INCH

FIG. 97.

50. Polar Diagrams of Velocities for Simple Plane Mechanisms.—The velocities in plane mechanisms can only be determined graphically from the positions of the virtual centres of the links when these centres fall within the limits of the drawing, and when their positions can be found with accuracy. Often the exact position of a virtual centre is difficult to define, because it lies at the intersection of two lines which make a very small angle with one another.

To avoid these difficulties, a general method of drawing diagrams for velocities and accelerations of points in mechanisms has been devised,* and a few simple cases will be considered here.

In Fig. 98 let ABC represent a rigid body having plane motion, and suppose the linear velocity v of the point A and the angular velocity ω of the whole body to be known. It is required to determine the linear velocities of the points B and C.

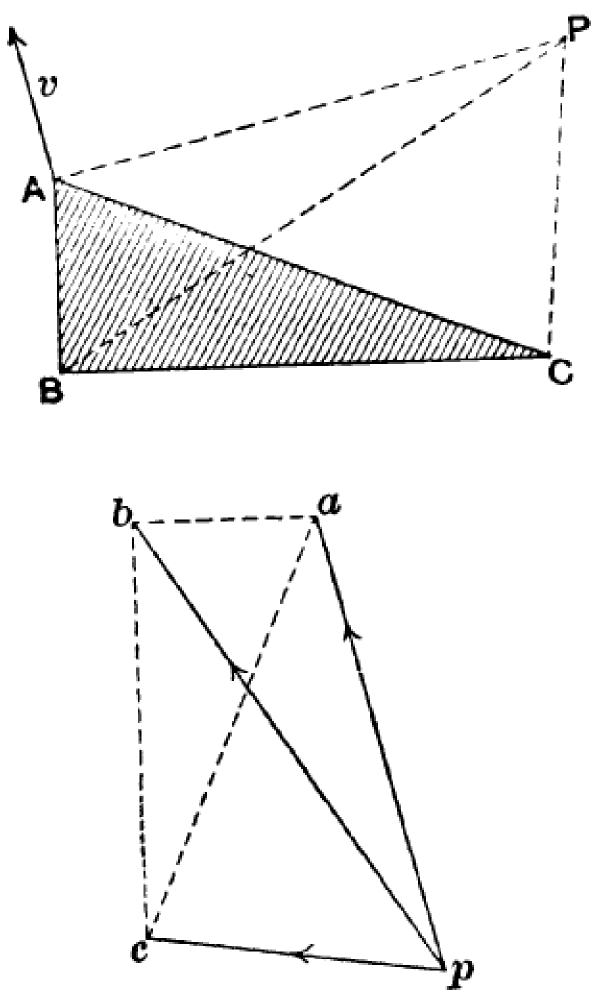

FIG. 98.

Let P be the virtual centre of ABC with regard to the plane of motion; then $AP=\frac{v}{\omega}$; hence the position of P can be found, since PA is perpendicular to the direction of v.

Join PB, PC. From these lines the directions of motion of B and C are known, and the magnitudes of the linear velocities are also known, since

$$\text{velocity of } B=\omega\times PB,$$

and

$$\text{velocity of } C=\omega\times PC.$$

* R. H. Smith, Graphics, Book I, Chap. IX; Burmester, Kinematik, Chaps. XI and XII.