

CHAPTER VII.

CONSTRAINT AND VELOCITY RATIO IN HIGHER PAIRING INVOLVING PLANE MOTION.

60. Constraint of Bodies having Plane Motion.—It has already been stated that a body free to move in a plane possesses three degrees of freedom and has three degrees of constraint. Further constraint may be applied by causing such a body to touch certain points on the surface of a second rigid fixed body, these points being known as points of restraint. A *point of restraint* of a figure or body may be defined as a point on its outline, so touched by a point on the outline of a second fixed figure or body, that no relative sliding motion is possible along or parallel to the common normal to the two figures at the point of contact. When thus restrained the body or figure is considered as being kept in contact with the point or points of restraint.

We may take an example to illustrate the meaning of this definition, and to show the actual nature of points of restraint. Suppose (in Fig. 116*a*) that it is required to arrange a support or base, *a*, for a tripod, *b*, so that an instrument fixed on *b* can be removed from its support and replaced exactly in its previous position. This may be effected by providing *b* with three rounded points or legs, *CDE*. A hole, *F*, is made in the base, *a*, and is of pyramidal or conical form, so that if the rounded end of *C* is placed in *F*, there will be contact at three points of restraint; in this way, so long as the contact is maintained, the only possible relative motion of *c* and *a* will be one of rotation about some axis

passing through the centre of the spherical surface of the end of C . The next step is to provide on a a slot or groove, G , of triangular cross-section as shown; when D is placed in this groove there will be two more points of restraint, and the only possible relative motion remaining will be a rotation about the axis CD . Finally the position of b is fixed relatively to a if the third point E is made to rest in contact with a flat surface, H , formed on or connected with a , thus furnishing the sixth point of restraint required (see § 7).

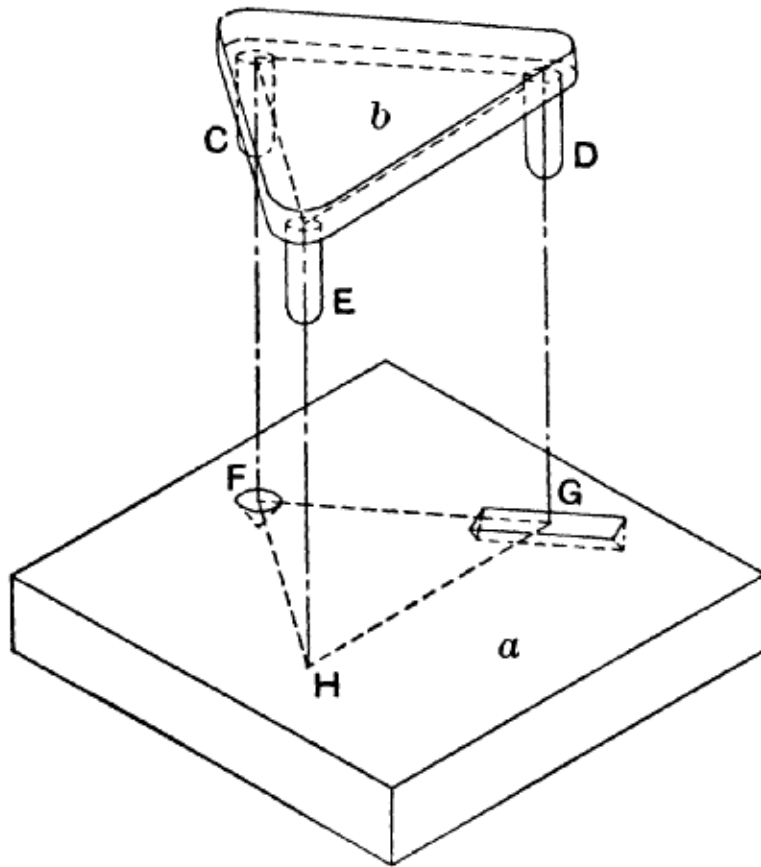


FIG. 116a.

The whole device is known as the "hole, slot, and plane."

The application of similar principles is illustrated in the design of Ewing's extensometer,* an instrument for measuring the deformation of test-pieces under stress. In this apparatus the bar or test-piece whose extension or compression is to be measured (a in Fig. 116b) carries a clip, b , attached by the points of two set-screws in such a way that b can move relatively to a about the axis of the set-screws at B . The clip b carries a projection, b' , ending in a rounded

* Ewing, Strength of Materials, p. 75.

Similar methods are followed in designing the so-called kinematic clamps and kinematic slides.*

A *kinematic clamp* is a contrivance intended to fix completely the position of one body with reference to another; a *kinematic slide* permits one body to have one degree of freedom with reference to another.

On consideration it is plain that in a kinematic clamp or slide the points of restraint must be suitably placed with regard to the shape of the body to be restrained.

It is thus proper to inquire what must be the disposition of the points of restraint required, either to define the position of one body relatively to another, or to permit the movable body to retain one degree of freedom, and thus to constrain its motion completely. We shall suppose that the movable body at first possesses three degrees of freedom, and is capable of plane motion.

Let a (Fig. 117) be such a body, and let a fourth point

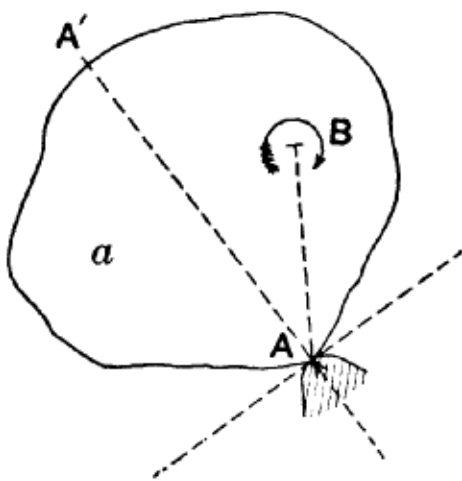


FIG. 117.

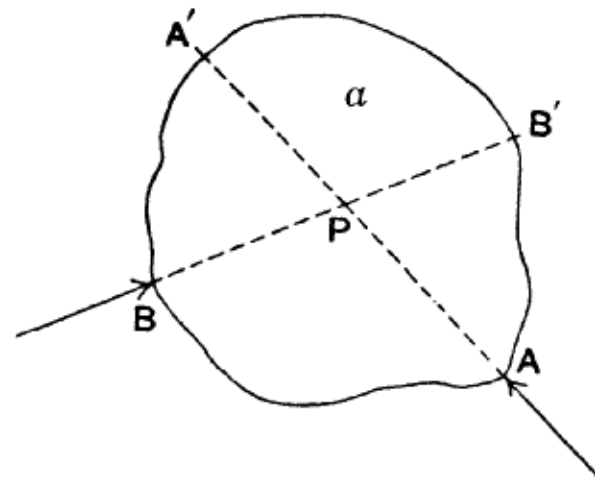


FIG. 118.

of restraint, A , be provided, in addition to the three points necessary for insuring plane motion. The arrow-head then represents the fourth point of contact of the restraining or fixed body.

Draw AA' normal to the tangent of the outline of a at A .

Any possible motion of a may be regarded as an instan-

* For an example of a kinematic slide, see Min. Proc. Inst. C. E., Vol. CXXXII, p 49.

taneous rotation about a virtual axis perpendicular to the plane of motion. We need therefore only consider how the single point of restraint, A , affects the possibility of turning the body a about such an axis. It must be remembered that by the definition of a point of restraint, a is to be kept in contact with the restraining body. This is impossible if the virtual centre is not somewhere along AA' , for if the virtual centre were, say, at B , a point about which only right-handed rotation is possible, it is plain that such rotation could only occur if the point A ceased to touch the restraining body. Hence we see that any possible instantaneous motion of a must be about a virtual centre situated in AA' , and any motion of translation must be along a line at right angles to AA' .

Next consider the effect of keeping the body a in contact with a restraining body at two points, A and B . Let the normals AA' , BB' intersect at P . The body is then only capable of an instantaneous rotation about P . If the normals are parallel, then only an instantaneous motion of translation, i.e., rotation about an infinitely distant axis, will be possible.

On adding another point of restraint, C (Fig. 119), it will be found that if we suppose that the body a remains in contact with the three new points of restraint, A , B , C , no movement is possible, except when the three normals intersect in one point or are parallel. In these cases instantaneous turning about the point of intersection, and instantaneous translation about an infinitely distant axis, are respectively possible, so that a at the instant considered will thus possess one degree of freedom and will have constrained motion.

In Fig. 119 a little consideration shows that no movement at all is possible except about an axis situated within the triangle PQR , so long as the restraining body is rigid. The whole field of motion, with the exception of PQR , then

becomes what Reuleaux calls a "field of restraint." But if movement did occur about an axis placed within the triangle PQR (in the figure such rotation could only be right-handed), the body a would at once cease to touch the restraining points with which we suppose it to be kept in contact. A similar result will be found with other arrangements of the points of restraint, and therefore in general

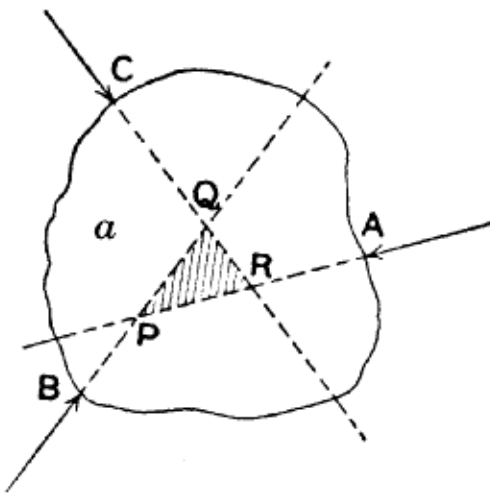


FIG. 119.

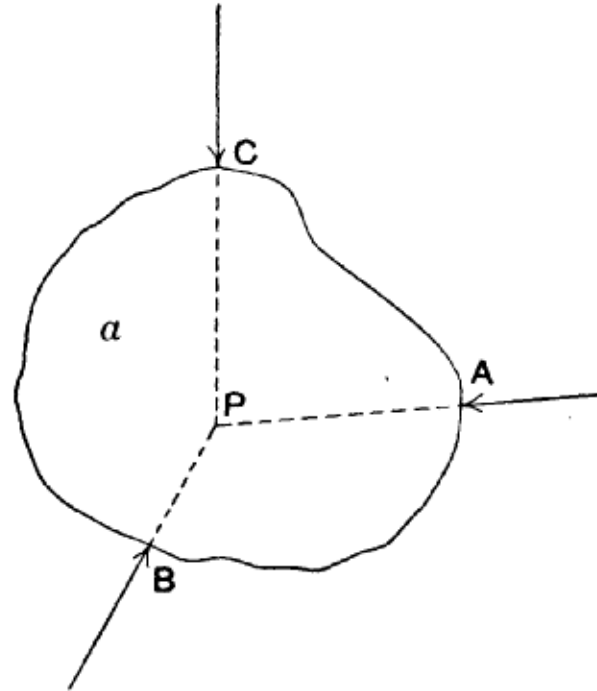


FIG. 120.

the position of a will be fixed if it is made to touch the restraining body at the three additional points A , B , C , a result already stated in § 7.

Fig. 120 shows the case in which the three normals meet in a point, P . If the shape of the body a is such that no point of restraint can be so applied as to have a normal that does not pass through P , then the body cannot be fixed by the application of three or any number of points of restraint, and its shape must be altered for that purpose. For example, a circular disc having plane motion could not be so fixed.

It is thus evident (a) that if one of two rigid bodies capable of relative plane motion remains in continuous contact with two points of restraint formed on the second body, the relative motion is constrained, and the virtual centre of the two bodies is always at the intersection of the two common normals.

