

CHAPTER VIII.

WHEEL-TRAINS AND MECHANISMS CONTAINING THEM. CAMS.

68. **Simple and Compound Wheel-trains.**—The determination of the velocity ratio in such a wheel-train as that of Fig. 130 involves no difficulty, for it is plain that one or

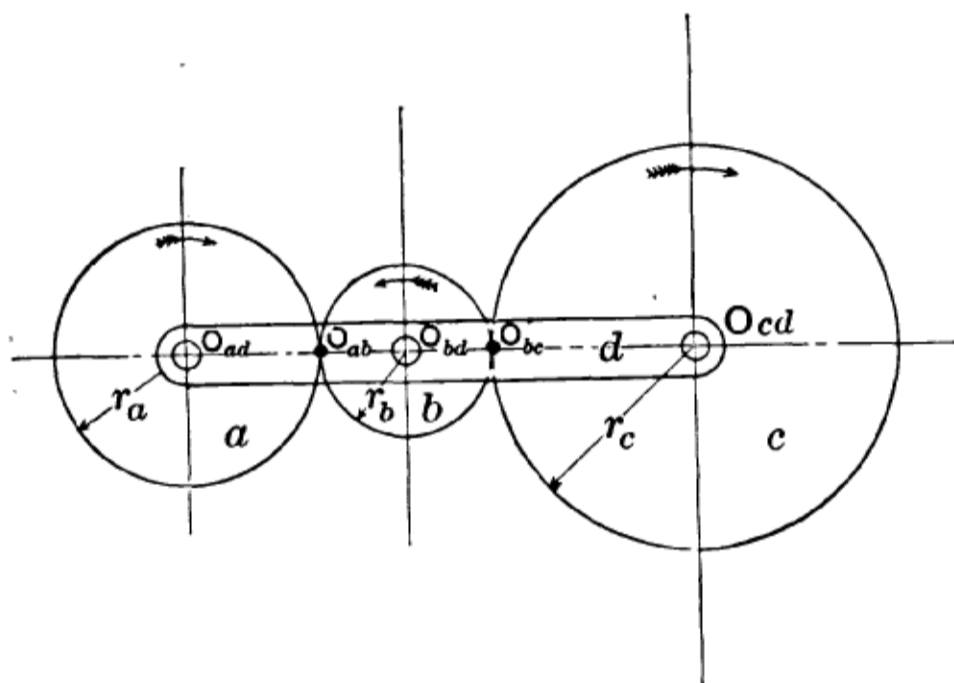


FIG. 130.

more intermediate wheels (as *b*) will not affect the numerical value of the velocity ratio of the first and last wheels. The linear velocity of the pitch-line of every wheel is the same, and the angular velocity ratio of the first and last, therefore, only depends on their own diameters, so that $\frac{\omega_{ad}}{\omega_{cd}} = \pm \frac{r_c}{r_a}$,

the sign depending on the number of idle wheels. Intermediate or *idle* wheels thus simply reverse the direction of motion. When all the wheels in the train have external contact, the angular velocity ratio of the first wheel to the last has a positive value (or, both wheels turn in the same

sense) if the number of axes is odd, while an even number of axes gives the velocity ratio a negative value. More complex wheel-trains, however, require further consideration. In Fig. 131 we have a compound spur-wheel mechanism of

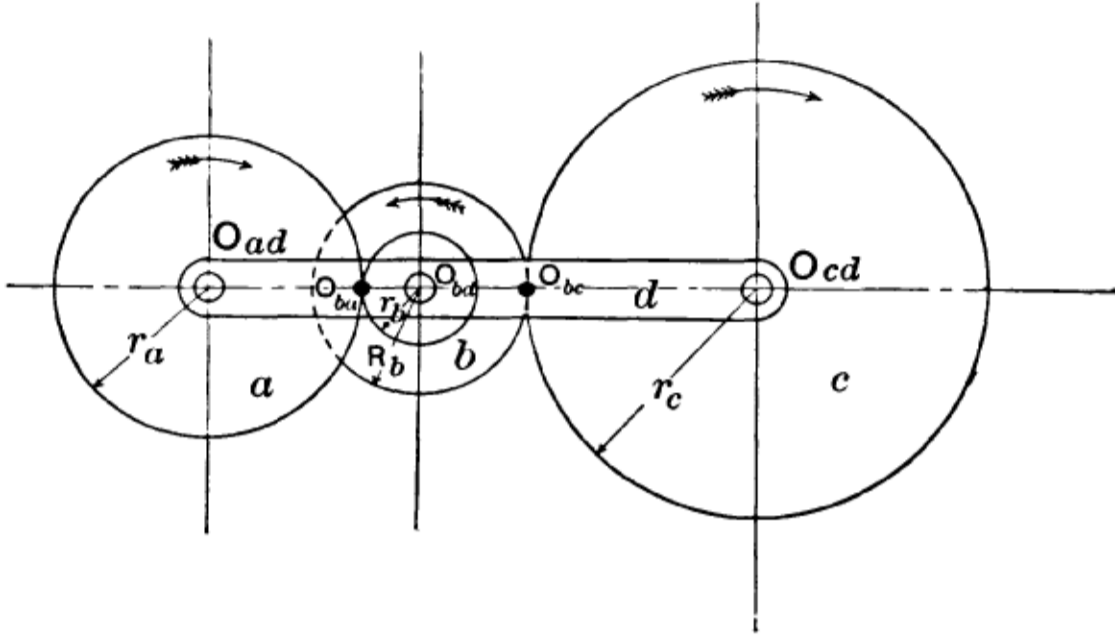


FIG. 131.

four links, d being fixed, while b consists of two wheels rigidly connected and turning on the same axis.

Let r_a , r_b , R_b , r_c , be the radii of the pitch-circles, then from § 64 we have

$$\frac{\omega_{ad}}{\omega_{bd}} = \frac{r_b}{r_a}.$$

Also,

$$\frac{\omega_{bd}}{\omega_{cd}} = \frac{r_c}{R_b}.$$

Hence

$$\frac{\omega_{ad}}{\omega_{cd}} = \frac{r_b \times r_c}{r_a \times R_b} = N.$$

Suppose a to be the driving-wheel, while c is the driven one; we see that the above result may be expressed by saying that

$$\begin{aligned} \text{velocity ratio} &= \frac{\text{revolutions of driving-wheel}}{\text{revolutions of driven wheel}} \\ &= \frac{\text{product of radii of followers}}{\text{product of radii of drivers}}. \end{aligned}$$

Instead of radii we might evidently put numbers of teeth.

It would be easy to find a single pair of wheels having the same velocity ratio as the given train. For example, if we had a pair of wheels, *A* and *C*, such that

$$r_A - r_C = r_a + r_b + R_b + r_c \quad \text{and} \quad \frac{r_A}{r_C} = \frac{r_a \times R_b}{r_b \times r_c},$$

these would have the same velocity ratio and the same distance from centre to centre. The point of contact of their pitch-circles would divide the distance $O_{ad} O_{cd}$ externally in the proportion of the angular velocities of *a* and *c*, and would in fact be the point O_{ac} .* Hence in Fig. 131 we have only to divide the line of centres, graphically or otherwise, in the proper ratio to find the sixth virtual centre.

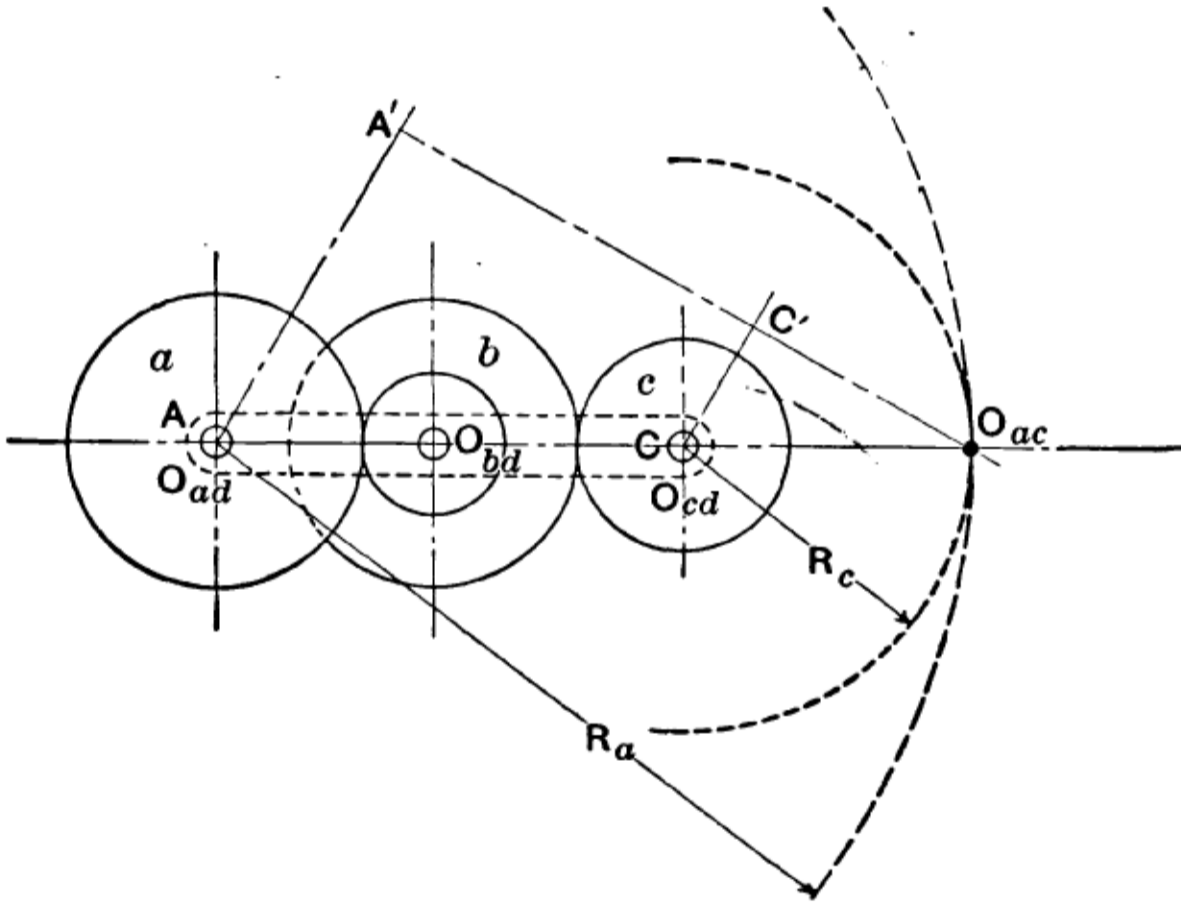


FIG. 132.

In doing this (as in working all problems connected with wheel trains), note must be taken of the sign of the velocity ratio, which depends on the presence or absence of *annular wheels* (i.e., wheels having internal contact), or of idle wheels, and also on the number of axes in the train. Take, for example, the two trains shown in Figs. 132 and 133, in

* For graphic methods of determining virtual centres of wheel-trains, see Kennedy, *Mech. of Machinery*, note to Chapter VI.

the first of which suppose $r_a = 2$, $r_b = 1$, $R_b = 2$, $r_c = 1.5$, so that the velocity ratio in Fig. 132 has the value

$$N = \frac{\omega_{ad}}{\omega_{cd}} = + \frac{1.5 \times 1}{2 \times 2} = + \frac{3}{8}.$$

In Fig. 133 we have a simple train having exactly the same numerical value for its velocity ratio (since $\frac{r_c}{r_a} = -\frac{3}{8}$), but in this case the negative value must be adopted, since the wheels a and c turn in opposite senses. In Fig. 132 O_{ac} may be found by drawing $AA'CC'$ parallel to one another, and of lengths 8 and 3 respectively, to any convenient scale. The intersection of $A'C'$ with the line of centres fixes O_{ac} . Evidently the given train might be replaced by a pair of wheels of radii R_a and R_c , the larger being annular, having their centres at A and C , and their pitch-circles touching at O_{ac} , as shown by the dotted arcs. Again, in Fig. 133

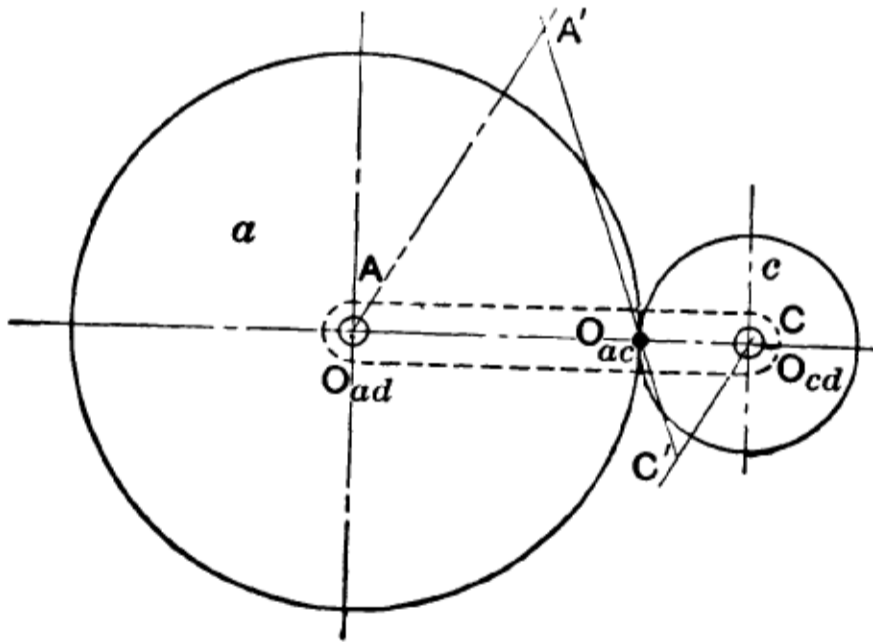


FIG. 133.

$AA'CC'$ must be drawn parallel, but in opposite senses, so as to allow for the negative velocity ratio, and O_{ac} is, of course, the point of intersection of AC and $A'C'$.

It is by no means necessary to have the centres of all the wheels of a train in one straight line. The back-gear of a lathe, for example, is an instance of a compound *reverted train* in which the centres of the first and last wheels coincide.

This arrangement makes no difference in the numerical value of the velocity ratio, and is simply adopted for convenience in construction.

• **69. Epicyclic Gearing.**—In the above examples of wheel-trains we have supposed the frame carrying the wheels to be the fixed link. Wheel gearing is often employed in which one of the wheels is the fixed link and the frame or arm carrying the remaining wheels is movable. Such gearing is called *epicyclic*, and we proceed to discuss some of its simpler cases.

We take first the mechanism of Fig. 133, but suppose *a* to be fixed, while *d* is rotated in a clockwise or positive sense (Fig. 134). Let *N* be the velocity ratio of the train, i.e., let

$$N = - \frac{r_a}{r_c} = \frac{\omega_{cd}}{\omega_{ad}}$$

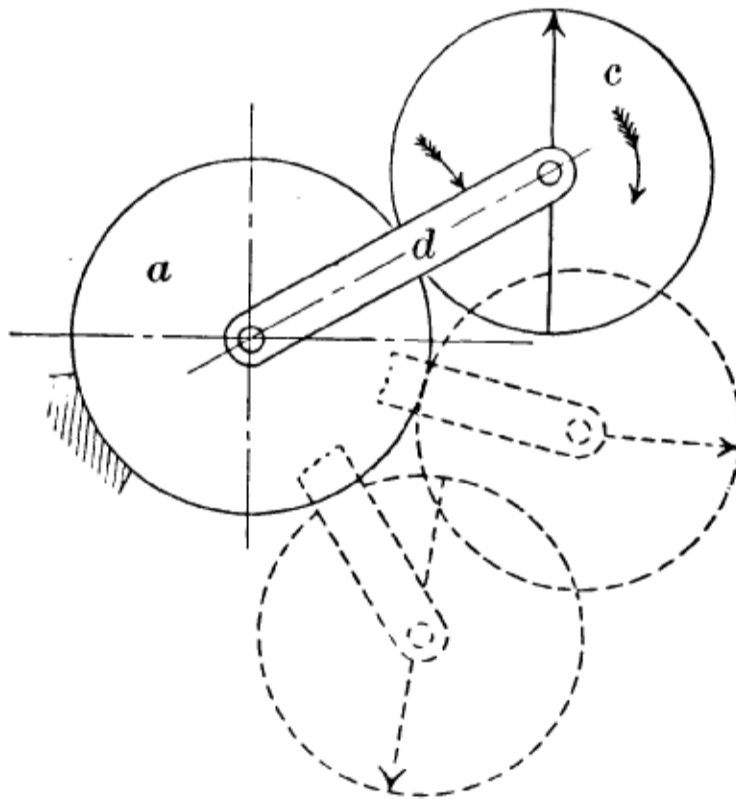


FIG. 134.

Plainly, if we consider ω_{da} as being positive in sign, then ω_{ad} must be negative, hence

$$\omega_{ad} = - \omega_{da}$$

Now in any case where two bodies, *c* and *d*, have motion relatively to a third, *a*, which is fixed, any angular movement of *c* relatively to *a* may be looked on as the algebraic

sum of the motions of c relatively to d and of d relatively to a . Thus

$$\begin{aligned}\omega_{ca} &= \omega_{cd} + \omega_{da} \\ &= +\omega_{cd} - \omega_{ad} \\ &= -\omega_{ad} \left(1 + \frac{r_a}{r_c} \right),\end{aligned}$$

or

$$\frac{\omega_{ca}}{\omega_{da}} = 1 - N,$$

where N is itself a negative quantity. A numerical example may, perhaps, make this clearer. Suppose the wheels a and c to have 100 teeth and 90 teeth respectively; these teeth have the same pitch, and we can, of course, take the ratio of the numbers of teeth instead of the ratio of the diameters or radii of the pitch-circles. Thus $\frac{r_a}{r_c} = \frac{100}{90}$; in other words, supposing d to be fixed, while a makes one revolution with

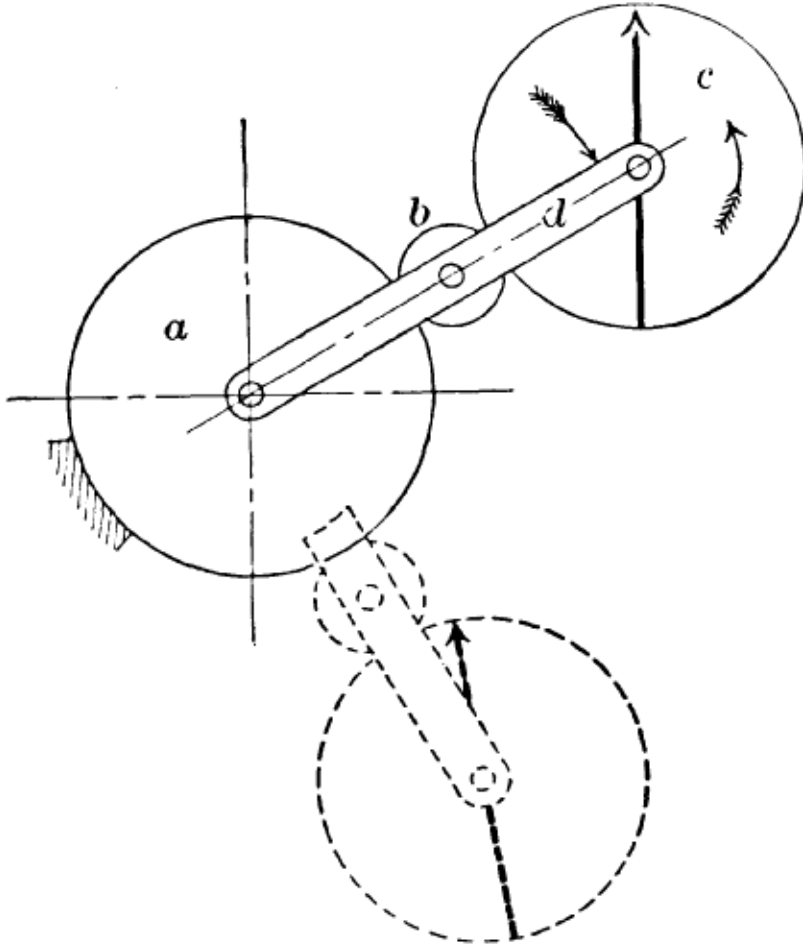


FIG. 135.

regard to d , c would make $1\frac{1}{9}$ in the opposite sense. Now suppose that in a certain time a makes -1 revolution, c making $+1\frac{1}{9}$, while d is at rest. Cause the whole mechanism to execute $+1$ rotation in the same time around O_{ad} ;

this brings a to rest, makes d perform $+1$ revolution, and therefore gives c $1\frac{1}{9} + 1 = 2\frac{1}{9}$ revolutions in the same sense as that of the arm.

If an idle wheel, b , had been interposed between a and c , as in Fig. 135, we should have had $N = +\frac{r_a}{r_c} = \frac{\omega_{cd}}{\omega_{ca}}$ a posi-

tive quantity, and $\omega_{ca} = \omega_{da} + \omega_{cd}$, whence $\frac{\omega_{ca}}{\omega_{da}} = 1 + \frac{\omega_{cd}}{\omega_{da}} = 1 - N$, as before. With the numbers of teeth, as in the example just given, and the train arranged as in Fig. 135, we should have, if $N = +1\frac{1}{9}$,

$$\frac{\omega_{ca}}{\omega_{da}} = 1 - 1\frac{1}{9} = -\frac{1}{9};$$

i.e., for each revolution of the arm, c makes $\frac{1}{9}$ revolution in the reverse sense.

Fig. 136 represents a compound epicyclic reverted train. Let $n_a, n_{b_1}, n_{b_2}, n_c$ be the numbers of teeth in the wheels

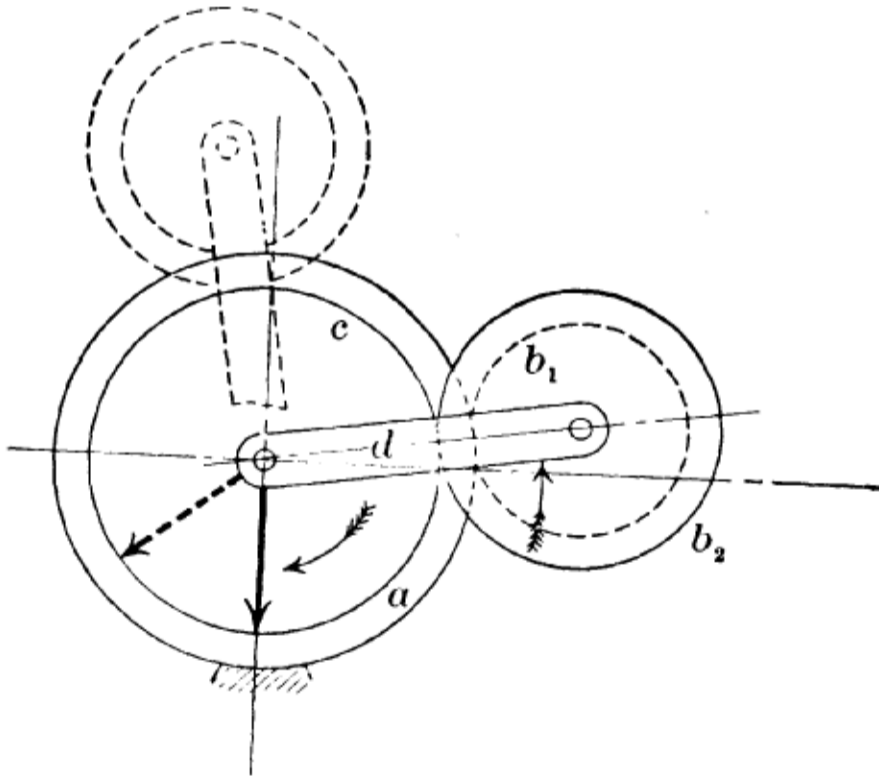


FIG. 136.

a, b_1, b_2 , and c respectively. Evidently, if the pitch of both pairs is the same, $n_a + n_{b_1} = n_{b_2} + n_c$. The velocity ratio of the train will be positive and has the value

$$N = \frac{\omega_{cd}}{\omega_{ad}} = \frac{n_a \times n_{b_2}}{n_{b_1} \times n_c}$$

hence $\omega_{cd} = -(N \times \omega_{da})$.

Further, when a is the fixed link

$$\omega_{ca} = \omega_{cd} + \omega_{da} = \omega_{da}(1 - N).$$

Thus, for instance, suppose a has 30 teeth, b_1 has 15, and b_2 and c have respectively 20 and 25; then

$$N = + \frac{30 \times 20}{15 \times 25} = + 1.6 = \frac{\omega_{cd}}{\omega_{da}},$$

and $\frac{\omega_{ca}}{\omega_{da}} = 1 - N = -0.6$.

Thus c will make 0.6 revolution for each revolution of the arm, but in the opposite sense. Such a train might evi-

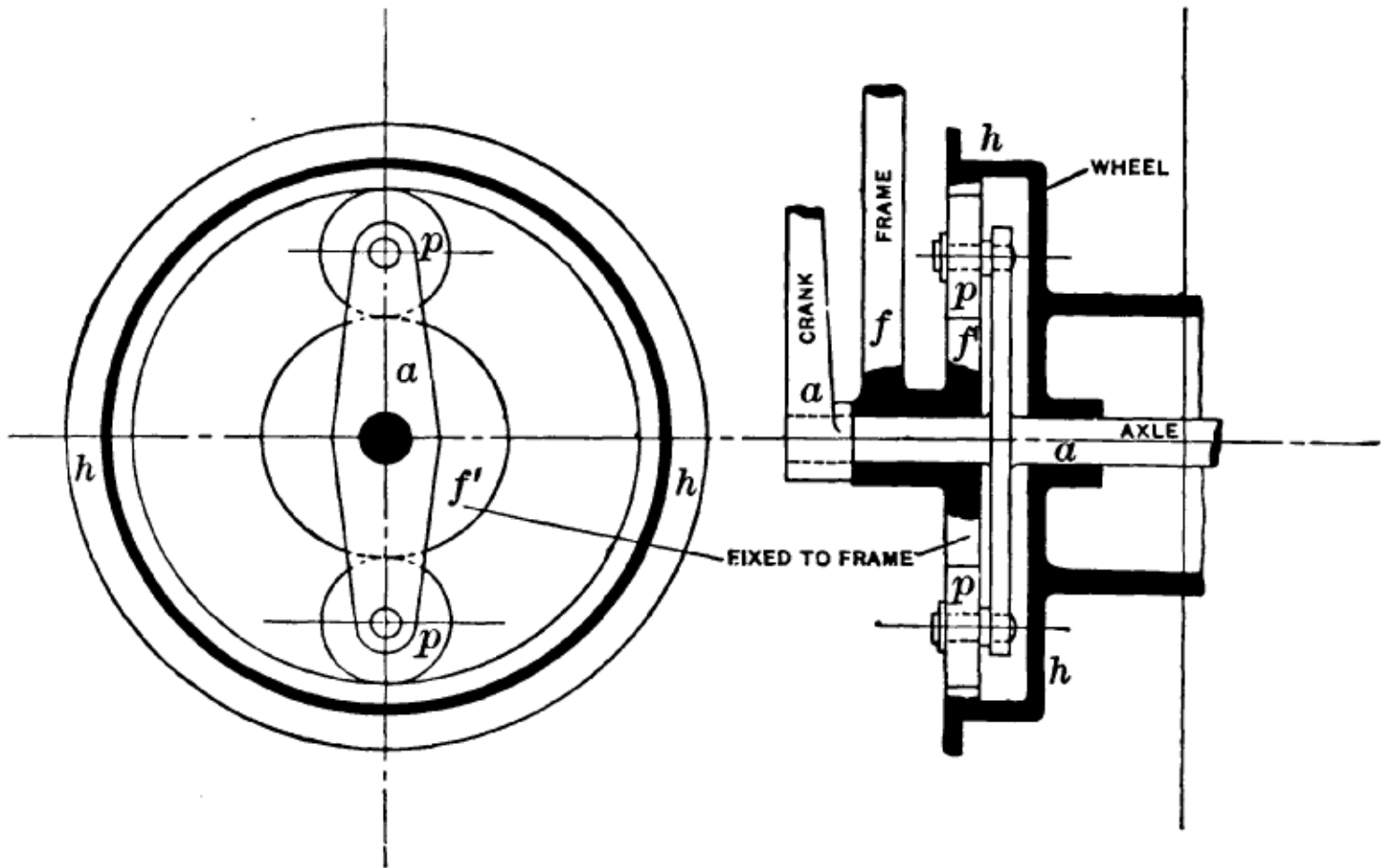


FIG. 137.

dently be arranged to give c a very slow rotary motion, say

$\frac{1}{10,000}$ revolution for each revolution of the arm d .

As an example of an epicyclic reverted gear containing an annular wheel the wheel-train used in certain front-driving bicycles* may be given. In Fig. 137 ff represents

* See also the Weston triplex pulley-block described in § 78, Chapter IX.