

## CHAPTER X.

### MECHANISMS INVOLVING NON-RIGID LINKS.

**79. Non-rigid Links.**—In giving a definition of a machine or of a mechanism we were careful to use the word “resistant” as applied to the material forming the links composing the mechanism. Many essential portions of actual machines are non-rigid, but are nevertheless resistant, and their occurrence, while it does little to complicate the machine from a kinematic point of view, often introduces dynamical problems of the greatest interest and difficulty. The different classes of non-rigid links, and pairs involving them, have already been noticed; we have now to study certain kinematic questions arising from their use.

In considering non-rigid links in mechanisms or machines it is necessary to take account of the way in which their form changes while in motion. One class of these links is composed of those which, while very yielding as far as bending or thrusting actions are concerned, do not change their length appreciably when a direct pull is applied. Belts, ropes, and chains, which come under this head, are therefore often of great use in machines where energy has to be transmitted in changing directions. This is usually done by causing the flexible tension-links, in the form of belts, ropes, or chains, to pair with, and communicate motion to, rotating drums or wheels. On account of their change of form, non-rigid links can have no virtual axes or virtual centres.

**80. Velocity Ratio in Belt-gearing. Length of Belts.**—The linear velocity of a rope or belt passing over two or

more pulleys may be considered for kinematic purposes as being the same throughout its length. In practice the stretching of a rope or belt under load often has an appreciable effect on the velocity ratio of the pulley it drives; we shall here treat questions of velocity ratio as if the belt or rope were inextensible. Fig. 163 represents a pair of cylindrical pulleys connected by a belt, which may be "open" or "crossed" so that the pulleys rotate either in the same or in opposite senses. We shall for the present neglect the effect of the thickness of the belt or rope.

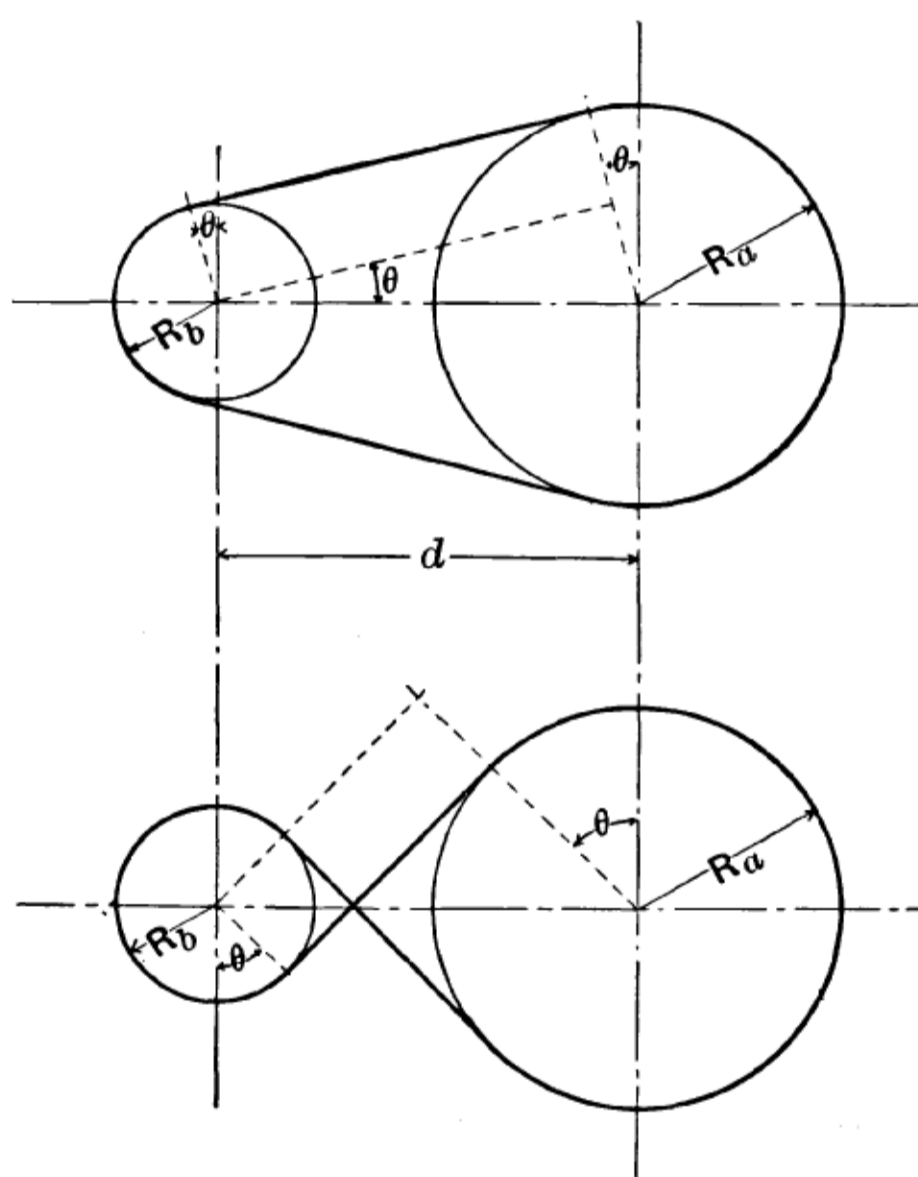


FIG. 163.

In these cases if  $V$  be the linear velocity of the belt and  $R_a, R_b$  the radii of the pulleys, the angular velocity ratio will evidently be found from the relation

$$\frac{\omega_a}{\omega_b} = \frac{V}{R_a} \cdot \frac{R_b}{V} = \pm \frac{R_b}{R_a},$$

the negative sign corresponding to the case of a crossed belt. It is, of course, assumed that there is no slipping.

The length of a belt is easily expressed in terms of the radii and the distance  $d$  between the centres of the pulleys. The total length of belt not in contact with the pulleys is

$$2\sqrt{d^2 - (R_a \pm R_b)^2},$$

the negative sign here corresponding to the case of an open belt. If  $\theta$  be the angle that the straight part of the belt makes with the centre line of the pulleys, then the length of belt in contact with the pulleys will be

$$(\pi + 2\theta)(R_a + R_b) \text{ for a crossed belt}$$

and  $(\pi + 2\theta)R_a + (\pi - 2\theta)R_b,$

or  $\pi(R_a + R_b) + 2\theta(R_a - R_b)$  for an open belt,

where  $\theta = \sin^{-1} \frac{R_a \pm R_b}{d}.$

The expression for the total length of belt will then be for an open belt

$$2\sqrt{d^2 - (R_a - R_b)^2} + \pi(R_a + R_b) + 2(R_a - R_b) \sin^{-1} \frac{R_a - R_b}{d},$$

and for a crossed belt

$$2\sqrt{d^2 - (R_a + R_b)^2} + (R_a + R_b) \left( \pi + 2 \sin^{-1} \frac{R_a + R_b}{d} \right).$$

It will be seen that the length of a crossed belt is thus constant so long as the sum of the radii and the distance between the centres of pulleys are constant quantities.

**81. Belt-gearing for Variable Velocity Ratio.** — Fig. 164 shows the arrangement of "cone pulleys" employed in driving machinery so as to render it possible, by shifting a

belt from one pair of steps to another, to obtain at will any one of several velocity ratios. It is plain that the same *crossed* belt will run with the same tightness on any pair of steps so long as the sum of the radii of each pair is the same. An open belt, however, is generally required, in which case the tension will be different on each pair of steps, unless

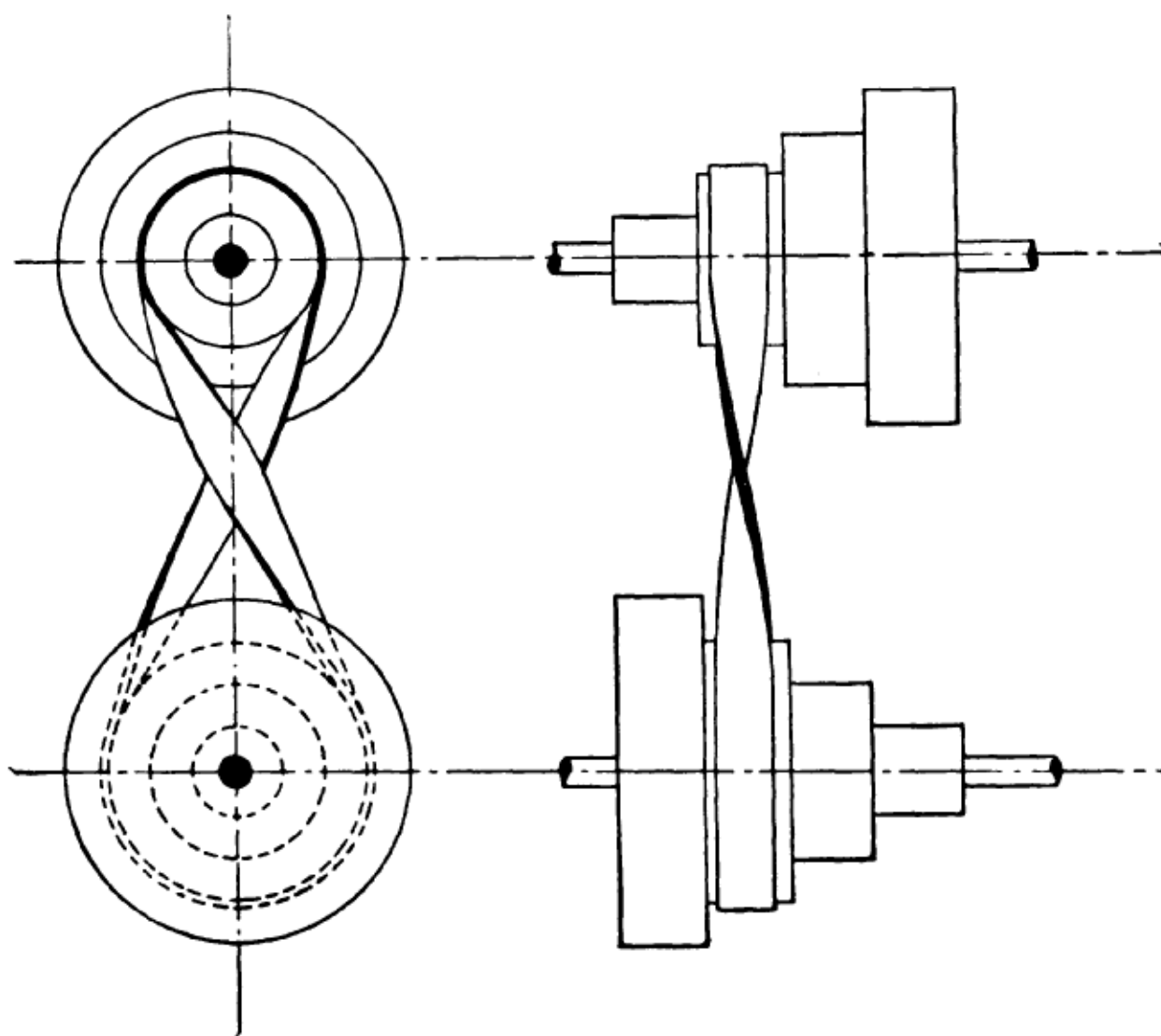


FIG. 164.

their diameters are specially calculated. Approximate methods for readily doing this have been devised,\* while Reuleaux † gives a rigorous graphical treatment of the problem. Referring to Fig. 163, we have as an expression for the length of an open belt

$$l = 2 \left[ d \cos \theta + \frac{\pi}{2} (R_a + R_b) + \theta (R_a - R_b) \right].$$

\* Unwin, Machine Design, Vol. I, p. 373 ; Smith, Trans. Am. Soc. M. E., Vol. X, p. 269.

† Reuleaux, The Constructor. Trans. by Suplee, p. 189.



and therefore  $KV = \frac{d}{\pi}(\cos \theta + \theta \sin \theta)$ .

Again, if we set off  $AL = \frac{l}{2}$ , and draw  $LM$  parallel to  $AO$ , and cutting  $AB$  in  $M$ , we have

$$LM = \frac{l}{2\pi}.$$

Draw  $MN$  parallel to  $BO$  and cutting  $HK$  in  $N$ , then

$$\begin{aligned} VN &= KN - KV \\ &= \frac{l}{2\pi} - \frac{d}{\pi}(\cos \theta + \theta \sin \theta). \end{aligned}$$

To obtain the value of  $\frac{d \sin \theta}{2}$  we need only draw a semi-circle  $OQP$  having a diameter  $\frac{d}{2}$ ; then

$$OQ = \frac{d \sin \theta}{2}.$$

Finally a curve  $DRST$  may be drawn by setting off  $VR = VS = OQ$ , and repeating the construction as required. This gives

$$\begin{aligned} NR &= VN + VR \\ &= \frac{l}{2\pi} - \frac{d}{\pi}(\cos \theta + \theta \sin \theta) + \frac{d \sin \theta}{2} \\ &= R_a, \end{aligned}$$

and

$$\begin{aligned} NS &= VN - VS \\ &= \frac{l}{2\pi} - \frac{d}{\pi}(\cos \theta + \theta \sin \theta) - \frac{d \sin \theta}{2} \\ &= R_b. \end{aligned}$$

Thus  $R_a - R_b = VR + VS = SR$ .

Plainly for given values of  $l$  and  $d$  we can determine  $R_a$  and  $R_b$  for any value of  $\theta$  (or for any required velocity ratio) by the aid of the curve  $DRST$ .

In practice it is usual to find that the diameters of the

first pair of steps or their radii,  $R_a$  and  $R_b$ , are given, together with  $d$ , the distance between centres of pulleys. The problem then is to find the radii of another pair of pulleys,  $R'_a$  and  $R'_b$ , on which an open belt of the same length will run with a given velocity ratio  $\frac{R'_a}{R'_b}$ . The author has found the following a convenient method of utilizing the Reuleaux diagram for solving this problem, and for finding incidentally the length of belt required. This length, however, is not often necessary, as it is more easily measured from the pulleys when finished and in position.

Draw the rectangle  $AOB$  and the curve  $DRST$  exactly as described above, and as shown in Fig. 165, making  $OA$ , say, 10 inches in length. This diagram can be used for finding pairs of radii of steps having any desired velocity ratio, and the lengths of these radii will be obtained in terms of  $d$ , the distance between the shaft centres. Having expressed  $R_a$  and  $R_b$ , the given pair of radii, in terms of  $d$ , it is easy, by applying a scale of inches and hundredths to the diagram, to determine that position of the line  $SR$  which will give the proper value to  $R_a - R_b$ . The length  $RN$  is then measured to the proper scale and the point  $N$  found. If required, the half length of the belt is then settled by drawing the line  $NM$ , and the next thing is to find another set of points  $R', S', N'$  such that  $R'N'$  and  $S'N'$  will have the

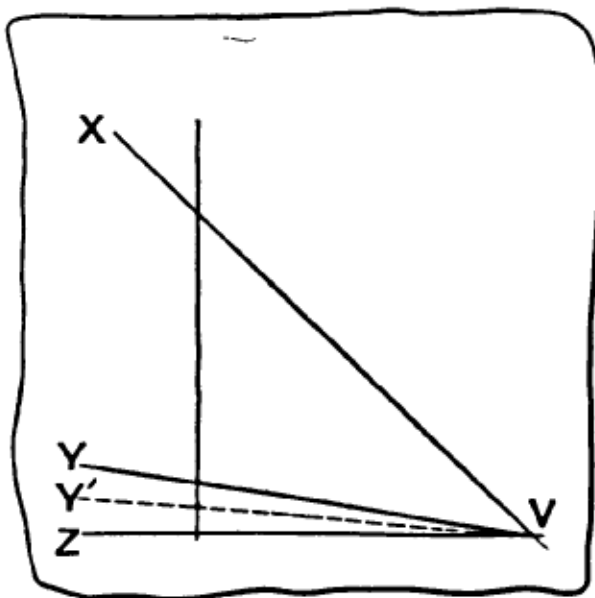


FIG. 165a.

ratio required for the radii of the next pair of steps. This is readily done by drawing on tracing-paper a set of radiating lines (Fig. 165a),  $VX$ ,  $VY'$ ,  $VZ$ , arranged so as to cut all lines perpendicular to  $VZ$  in the required ratio, namely,  $\frac{R'_a}{R'_b}$ . It

is convenient to draw another line,  $VY$ , such that lines perpendicular to  $VZ$  are also cut in

