CHAPTER II.

PLANE MOTION.

§ 4. RELATIVE POSITION IN A PLANE.

WE have defined motion, so far as we are now studying it, as change of position. We have seen also that we have to consider only the change in the position of one body relatively to another, and not the absolute motions of bodies. We shall now commence the more detailed treatment of this branch of our subject.

It is necessary first to examine the general conditions by which *relative position* is or may be determined. Just as the absolute *motion* of a body in space is a matter which does not concern us, so the absolute *position* of a body in space or of a figure in a plane is indifferent to us. We can assume a point or a figure stationary in any part of the plane, our object is solely to examine the position of others relatively to it.

Starting then with the notion of a fixed point in a plane, we have first the proposition that **the position of one point relatively to another is determined solely by the distance between them.** It is entirely unaffected by the *position* of the line joining them. Thus in Fig. 8, the points A and A_1, which are at the same distance from P, have the same position relatively to it, and, generally, all points in a circle occupy the same position relatively to its centre for the same reason. A point having no angular magnitude, that is, no *sides*, there cannot be any differences of angular position relatively to it. It is evident, however, that the points AA_1

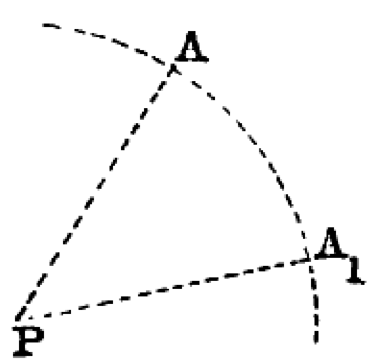

FIG. 8.

&c., occupy different positions in or relatively to the plane in which they are. We see therefore that **the position of a point in a plane is not determined by its position relatively to a point in that plane.**

A *line* is fully determined if two of its points be known. **The position of a line relatively to a point is therefore known if the positions of two of its points relatively to the fixed point be known.** These

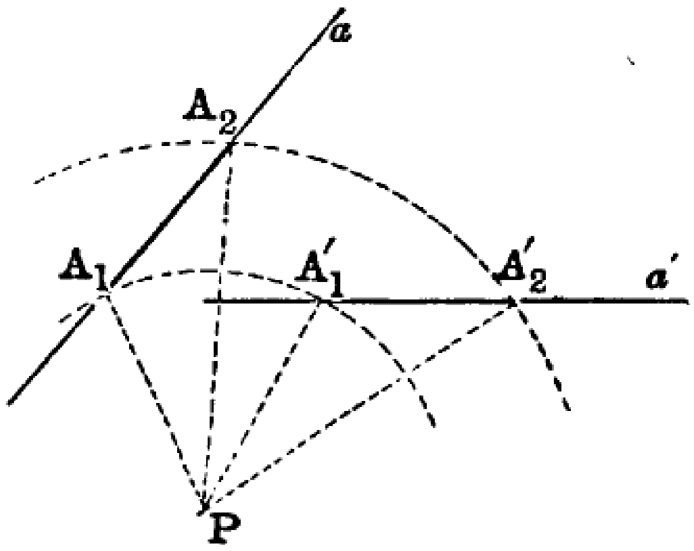

FIG. 9.

positions are determined, as mentioned in the last paragraph, solely by distances from the fixed point. As long as these distances are the same the position of the line relatively to the point is the same also. Thus in Fig. 9, where $A_1P = A'_1P$

and $A_2P=A'_2P$, the position of the line A_1A_2 relatively to the point P is the same as that of $A'_1A'_2$ relatively to the same point. But these lines are in different positions in the plane—hence **the position of a line relatively to a plane is not determined by its position relatively to a point in the plane.**[1]

The position of a *point* relatively to a *line* may be determined in two ways. It is known (i) **if its distances from two points of the line be known,** (ii) **if the positions of the lines joining it to two points of the line be known.** Thus in Fig. 10 the position of the point A

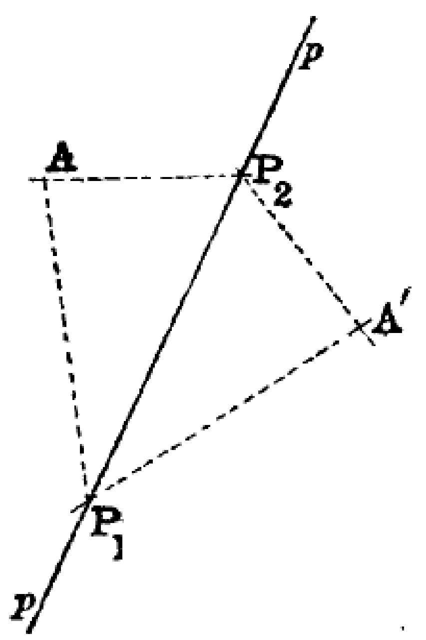

FIG. 10.

relatively to the line P_1P_2 is determined by (i), if the distances AP_1 and AP_2 be known, or by (ii) if the angles AP_1P_2 and AP_2P_1, made by AP_1 and AP_2 at the points P and P_2 of the line, be known. But we can always find two points in the plane, one on each side of the line, which shall satisfy any given conditions either in (i) or (ii). The point A', for instance occupies the same position relatively to P_1P_2 as A. A point may therefore occupy two

[1] It may be noticed in passing that the theorems just given are equally true whether or not all the points or lines are in the same plane. They hold good, that is, for spheric equally with plane motions.

positions in the plane for all positions which it can take relatively to any line in the plane, so that its position in the plane is not absolutely determined by its position relatively to a line in the plane.

We can, however, adopt some simple convention to distinguish between the two parts into which the line divides the plane; taking distances measured from P_1P_2 as positive to the one side and negative to the other, for instance. If we suppose this to be done, the symmetrical positions A and A' can be distinguished from each other, and the position of A in the plane is by this means determined when its position relatively to the line P_1P_2 is known.

The position of one line relatively to another in the same plane is known if the positions of two points in the first are known relatively to two points in the second. Here again we have an indeterminateness of the same kind as in the last case. A line may occupy two different positions in the plane, as A_1A_2 or $A'_1A'_2$ Fig. 11, and yet be in the same

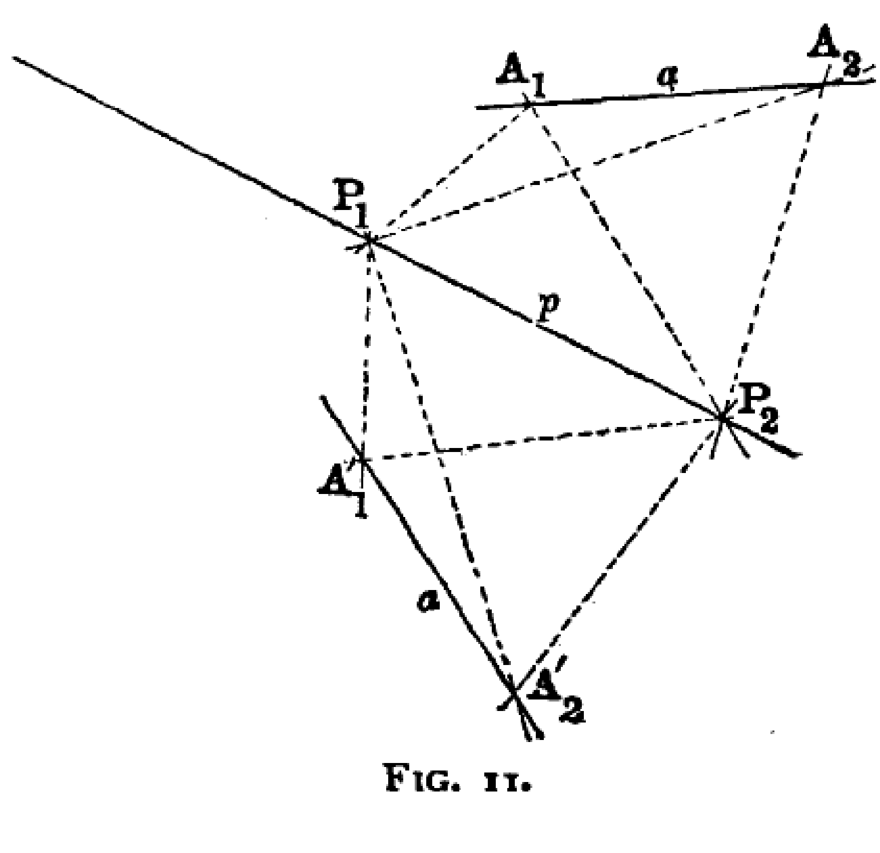

FIG. 11.

position relatively to a line P_1P_2 in the plane. If these positions be distinguished by such a convention as that just alluded to, however, the indeterminateness disappears, and we may say that **the position of a line in a plane is determined by its position relatively to any other line in the plane.**

If *a*, Fig. 12, be any given plane figure, and *A B* any two points in that figure, then if we know the positions of

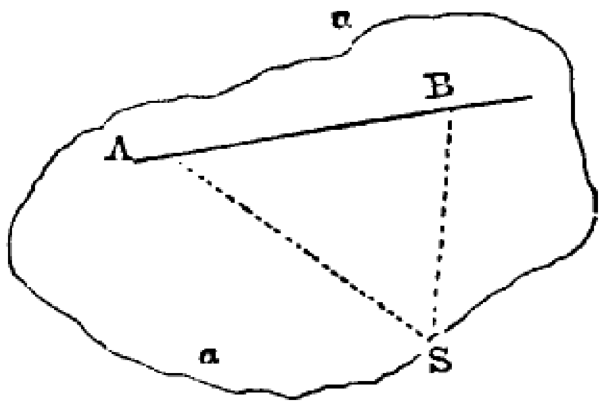

FIG. 12.

these two points we know the positions of all the others. For any other point, as *S*, can be found at once as the vertex of a triangle of which the magnitudes of all three sides (as *S A*, *A B*, *B S*) are known. **The position of a plane figure in a plane is therefore known if the positions of two points—that is, of a line—in it be known relatively to two points in the plane.**

If we discard the convention of positive and negative alluded to above, the position of a point in, *i.e.* relatively to, a plane is known only if its position relative to *three* other points in the plane, not in the same straight line, be known. Similarly the position of a line, and consequently of a plane figure, in a plane, is only completely determined if the positions of two of its points relatively to three points in the plane—not in the same straight line, be known. For our purposes, however, the two points will generally be sufficient, it is seldom that the circumstances of the case leave any doubt as to which of the two possible positions is the required one.

§ 5. RELATIVE MOTION IN A PLANE.

We have seen in the last section the conditions necessary to determine the relative positions of points, lines and figures in a plane. The *motion* of a point or line, however, is represented to us by the series of different *positions* which it occupies relatively to another point or line, &c. Each one of these is determined by the same conditions, so that the conditions which determine the *position* of the point or line relatively to any other, determine also its *motion* relatively to that other. We get therefore,—in most cases by little more than verbal alteration,—the following propositions as to relative motion in a plane, corresponding to those of the last section as to relative position.

One point can move relatively to another only along the line joining them. Thus in Fig. 13, A

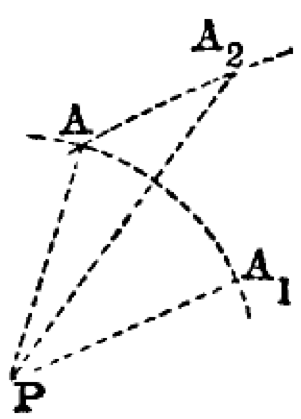

FIG. 13.

does not move relatively to P in moving to A_1, because every point in AA_1, its path of motion, has the same position relatively to P. In moving from A to A_2, however, A moves through the distance $PA_2—PA$ relatively to P.

The motion of a line relatively to a point is determined by the motion of two points in it relatively to that point. Each of these points can move, relatively to the fixed point, only along the line joining them. We see then that (just as in the case of position) the motion of a point or a line relatively to a plane is not determined by its motion relatively to a point in that plane. If a line turn about a point, for example, it remains stationary relatively to that point, although it is in continuous motion relatively to the plane.

The motion of a point relatively to a line is determined by its motion relatively to two points of the line.

The motion of a line relatively to a plane in which it moves (or to a line in that plane), is determined by the motions of two points in the one relatively to two points in the other.

And lastly **the motion of any plane figure relatively to its plane is determined by the motions of any two points,** *i.e.* **of a line, in it**

The last theorem may be stated also in another way. The figure being supposed rigid, no point in it can move relatively to any other,—all points in it, therefore, must have the same motion. But this motion is that of any line in it. When we have given, then, the motions of any two points whatever in a figure, we know the motion of the figure, and we know also that the motion of every other point in the figure is the *same* (in the sense already explained) as the known motions of the two arbitrary points with which we started.

We have already seen that when a body has plane motion the whole motion of the body is known when that of any plane section of it, moving in its own plane, is known :— the motion of the section or figure represents that of the whole body. But we have now seen further that the (plane) motion of a figure is known if the motions of two of its points be known. **The plane motion of a body,**