CHAPTER IV.

VIRTUAL MOTION IN MECHANISMS.

§ 12. DETERMINATION OF THE VIRTUAL CENTRE IN MECHANISMS.

WE have seen that in order to determine the virtual centre about which a body is moving at any instant, it is necessary and sufficient to know the direction of the motion of two points in the body at that instant. We must now consider this more in detail, in order particularly to apply our knowledge to the solution of the problem in the case of mechanisms.

The path in which a point is moving in the plane may be supposed given, either by its equation or by its form actually traced out on paper. In the former case the direction of motion, or tangent to the curve, can be calculated, and in the latter case it can be drawn. We have to deal exclusively with the latter case in our work. There are few cases in which it is at all difficult to draw the actual path in which any point of a mechanism is moving, and to construct a tangent to this path at any point, and no cases at all, so far as we know, in which it is not greatly more convenient to do so than to calculate an equation to that path. Finding the direction of motion of a figure then means, for us, simply drawing the paths of two of its points and constructing tangents to them, or of course (if possible) constructing the tangents without drawing the point-paths themselves, which are not, in most cases, of any direct importance to us.

We require to know the direction of motion of two points in the body. *Any* two points will serve, provided their virtual radii be not coincident, in which case, of course, they would not determine the intersection which we require. The problem, therefore, resolves itself simply into a choice of points, a matter which we must here examine briefly

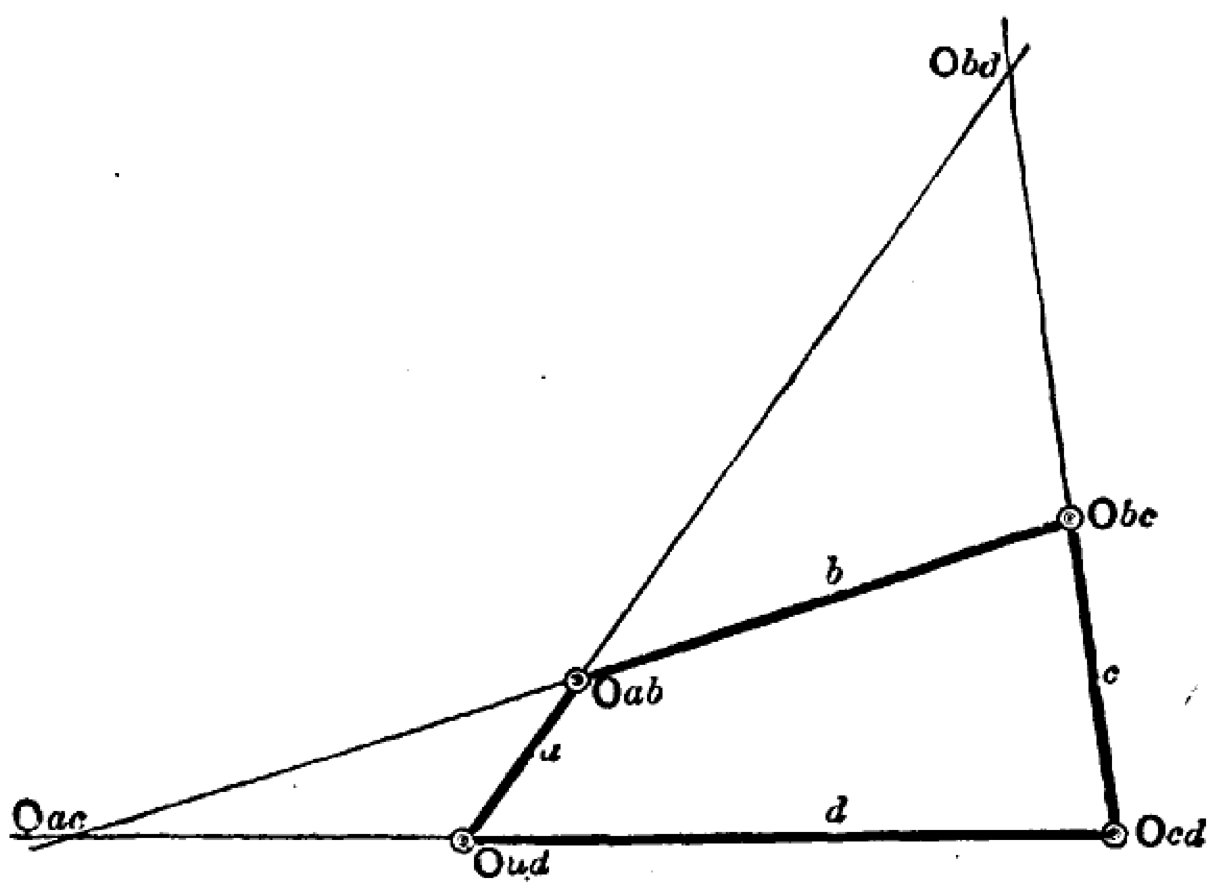

FIG. 29.

—we shall have numerous examples in succeeding chapters. To start with the simplest possible case, let it be required to find the virtual centre of every link relatively to every other in such a mechanism as Fig. 29, which consists of four links connected by four turning pairs. The axes of the pairs are all parallel, so that the links have only plane motion. Calling the links *a*, *b*, *c*, and *d*, we shall call their virtual centres O_{ab}, O_{bc}, etc., the suffixes denoting the links for which the particular point O is the virtual centre. The virtual centres of *adjacent* links are permanent centres, and are, as we have already seen, simply the centres of the pairs connecting them. Relatively to *d* for instance, every point in *a* moves always about the point O_{ad}, which is the centre point of the pair connecting *a* and *d*. By mere inspection therefore, we have at once the points O_{ab}, O_{bc}, O_{cd} and O_{da}, as the virtual (and permanent) centres of the four pairs of adjacent links. There are two other virtual centres in the mechanism, those for the two pairs of non-adjacent links; O_{ac} for the links *a* and *c*, and O_{bd} for the links *b* and *d*. We may take the latter first;—*b* is connected to *d*, and its motion constrained, by the links *a* and *c*, and we know the motion of every point in these two links relatively to *d*. But *b* has one point in common with *a*, viz. the point O_{ab} and also one point in common with *c*, the point O_{bc}. We know the motion of these points relatively to *d* as points of *a* and *c*, and of course they must have the same motion relatively to *d* as points of *b*, for they cannot have two different motions relatively to the same body at the same time. We have therefore at once the motion of two points in *b* relatively to *d*, which is all we require. O_{ab} is moving in a circle round O_{da}—without drawing its path then, or even constructing the tangent to it, we can at once draw its virtual radius, which is at right angles to the tangent, and which is simply the axis of the link *a*. In exactly the same way the axis of *c* is the virtual radius of O_{bc} and is at right angles to the direction in which it is moving. The virtual centre of *b* relatively to *d* is therefore at the join of these two axes, as shown by the fine lines in the figure. By the same reasoning it can be shown at once that the point O_{ac} is at the join of the axes of *b* and *d*.

Quite generally, therefore, in a chain such as Fig. 29, consisting of four links connected by four parallel turning pairs, **the virtual centre of either pair of opposite links is the join of the axes of the other pair; the virtual centre of any pair of adjacent links is the join of their own axes, and is a permanent centre.**

An inspection of Fig. 29 shows a rather remarkable regularity in the disposition of the virtual centres. The six centres lie in threes upon four lines, and the three centres on

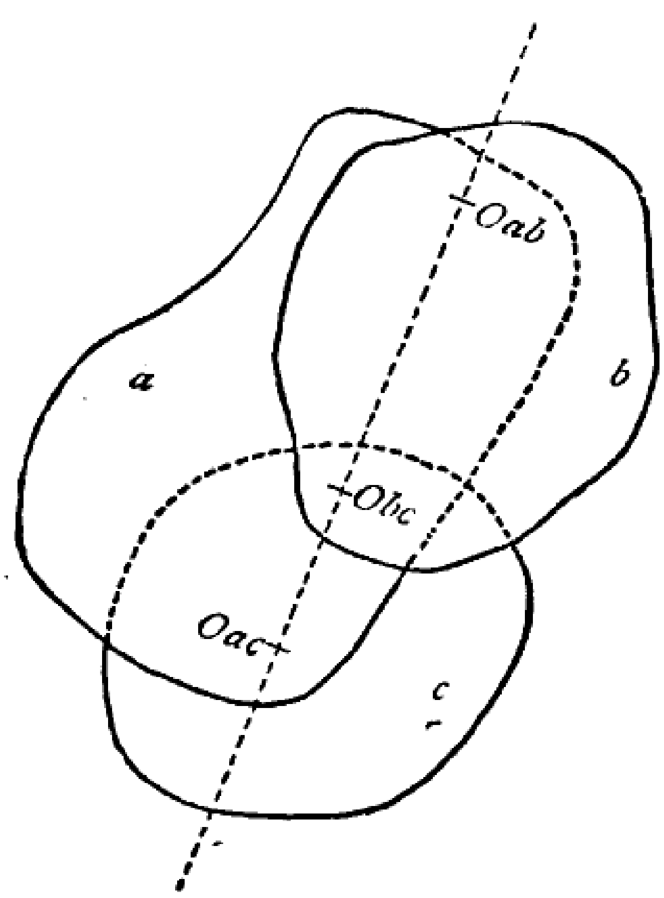

FIG. 30.

any one line are always those corresponding to three particular links out of the four. The links *a*, *b*, and *c*, for instance, give us the three virtual centres O_{ab}, O_{bc}, and O_{ac}, and these three points are in one line, here the axis of *b*. The links *b*, *c*, and *d*, similarly, give us the points O_{bc}, O_{cd}, and O_{bd}, and these again lie all on one line, here the axis of *c*. The question comes at once whether this is some mere coincidence, belonging to the very simple mechanism which we have chosen for illustration, or whether it represents some general law which we may apply in other cases. It is in fact quite general, and the proof is simple. Let a, b, and c (Fig. 30), be any three bodies whatever, having plane motion, and let O_{ab}, O_{bc}, and O_{ca} be the virtual centres for their motion. O_{ac} is a point both of a and of c; as the former it is moving about O_{ab} relatively to b, as the latter about O_{bc}. That is, its direction of motion as a point in a is at right angles to the line joining it to O_{ab}, and its direction of motion as a point in c is at right angles to the line joining it to O_{bc}. But it can have only one direction of motion relatively to b, whether it be treated as a point of a or of c, and as this direction is normal to both the lines just mentioned, they must either be parallel or coincident. They cannot be parallel, for they both pass though the same point O_{ac} on the paper—they must therefore coincide. The radius O_{ac} O_{ab} coincides with O_{ac} O_{bc}—the three points named therefore lie in one straight line. We might have started with O_{ab} or O_{bc} instead of O_{ac} and should always have come to precisely the same result, which may be summed up as follows; **If any three bodies a, b, and c, have plane motion, their three virtual centres O_{ab}, O_{bc}, and O_{ac} are three points upon one straight line.**[1]

We may now examine another simple chain, the one shown in Fig. 31, which is the same as one which we have already noticed. Using the same notation as before, we have again the points O_{ab}, O_{bc}, O_{cd}, and O_{ad}, the virtual centres of adjacent links, at once. Three of them are, as in the last case, the centres of turning pairs, and the fourth is the centre of the sliding pair c d, and therefore a point at infinity. All four, including, O_{cd} (see pp. 43 and 46),

[1] This proof, it may be noticed, is quite independent of the bodies being adjacent links in a mechanism, or indeed of their belonging to a mechanism at all; it applies to any constrained *plane* motion. For the corresponding theorem in spheric motion see § 63.

are permanent as well as instantaneous centres. The virtual centres of opposite links, O_{bd} and O_{ac}, are as easily found as in the last case, but the points which determine them are not, perhaps, quite so obvious. The link *b* has, as before, one point in common with each of the links *a* and *c*; we know the motion of every point in these links relatively to *d*, for we have found O_{ad} and O_{cd}, we therefore know the motion of two points in *b* relatively to *d*, these points being

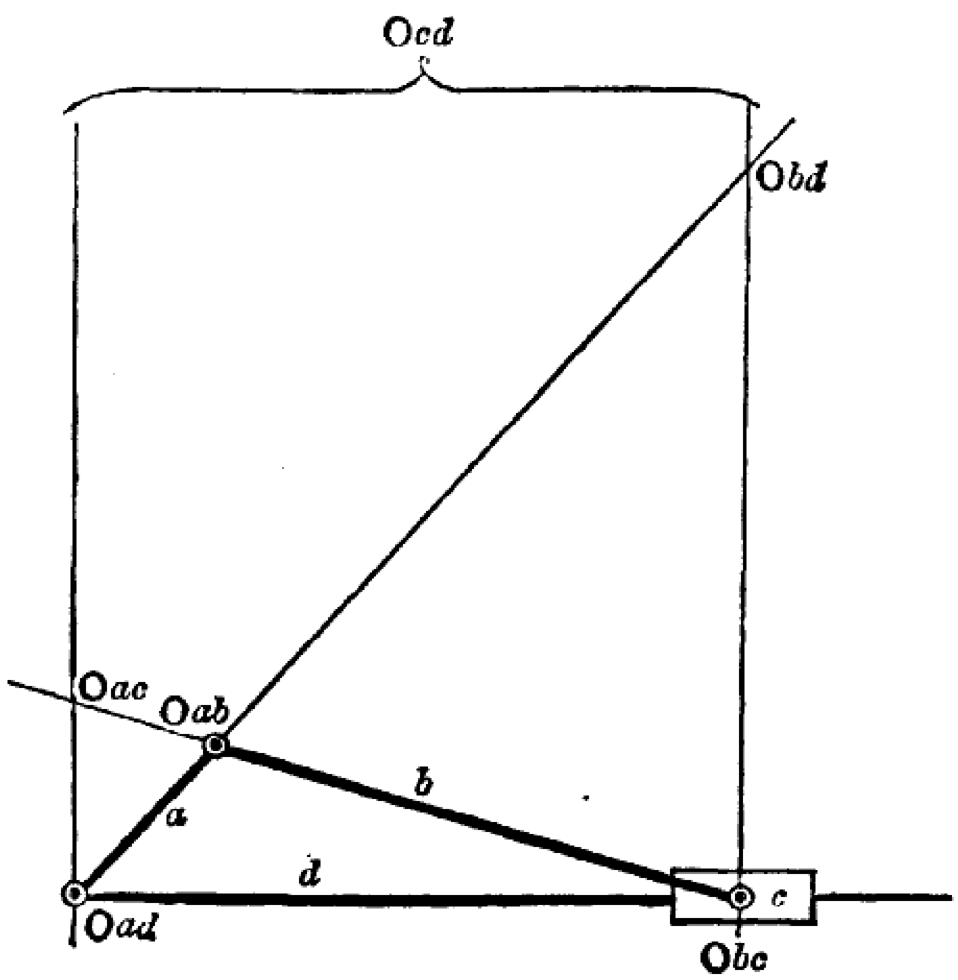

FIG. 31.

O_{ab} and O_{bc}. The virtual centre of *b* relatively to *d* is at the join of the virtual radii of these points, exactly as in the former case. The virtual radius of the point O_{ab} is the line joining that point to O_{ad}, or the axis of the link *a*. The virtual radius of O_{bc} is the line joining that point to O_{cd}, which is simply a line perpendicular to the axis of the sliding pair. The construction lines and the point O_{bd} at their join are shown in the figure.

By similar reasoning we get the point O_{ac}, the virtual centre of *a* relatively to *c*, but the reasoning is perhaps a little more difficult to follow. The link *c* has two points whose motion relatively to *a* we know, for it has one point in common with each of its adjacent links *b* and *d*, both of which are adjacent to *a*. O_{ac} must be, as before, the join of the virtual radii of these two points. The virtual radius of the one, O_{bc}, is the line joining it to O_{ab}, or simply the axis of the link *b*, and can at once be drawn. The virtual radius of the other, O_{cd}, is the line joining O_{ad} and O_{cd}. But O_{cd} is a point at an infinite distance, hence all lines passing through it are parallel on our paper (p. 43), so that to draw the line in question we have only to draw through O_{ad} a line parallel to the virtual radius of O_{bc} already constructed, (and therefore perpendicular to the axis of the sliding pair) and we have the line required, which gives us O_{ac} directly. The construction is shown on the figure.

By the help of the theorem about the virtual centres of three bodies which we proved above, this proof can be much shortened. From the fact that *a*, *b*, and *c* are three bodies having plane motion, we know that the point O_{ac} must lie on the line joining O_{ab} and O_{bc}, and similarly, considering the three bodies *a*, *c*, and *d*, we know that O_{ac} must lie on the line joining O_{ad} and O_{cd}. To find O_{ac} therefore, we have only to draw these lines to their join, which is just what we have done. Similar reasoning would, equally briefly, have given us the position of the point O_{bd}.

As these mechanisms are very important and will often be referred to, it may be well to use the name "lever-crank" for Fig. 29, and "slider-crank" for Fig. 31, the link *d* being supposed the fixed one in each case.

Fig. 32 shows a mechanism having a very close relationship to Fig. 31, but one in which it may appear at first sight