CHAPTER V.

RELATIVE VELOCITIES IN MECHANISMS.

§ 14. RELATIVE LINEAR VELOCITIES

WE have so far considered motion only as change of position, entirely without reference to the time occupied by the change, that is, to the velocity of the different points of the body while moving; and we have seen that there are many kinematic problems which can be treated entirely without consideration of velocity. Connected with velocity, however, there are two distinct sets of problems which we have to examine, and one of these we can now take up. The absolute velocity of any point in a machine, as well as the changes in that velocity, depend, as we shall see presently, upon the forces acting upon the different parts of the machine. With these we have not at present anything to do. But the *relative velocities* of different points in the machine at any given instant can be determined by purely geometric considerations, so that we have already the means of dealing with them. We have seen that at each instant every body[1] in a machine or mechanism is virtually turning about some particular point, and have seen, further, how to find that point. **Every link of the machine, therefore, is simply in the condition of a wheel turning**

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[1] Limiting ourselves to *plane motion ;* see end of § 2.

about its axis, or a lever vibrating on its fulcrum, and this no matter how complex in appearance, or even in reality, the connection between the different parts of the machine may be. But in such a case it is obvious that the velocities of the different points must be simply proportional to their distance from the centre of rotation, that is proportional to their real, or virtual, radii or "leverage." The velocity of any one point being then known, the determination of the velocities of the others becomes a mere matter of finding the virtual centre and the distances of the various points from it. And even without knowing the *absolute* velocity of any point the same method gives us the *proportionate* velocities of all the points, quite independently of their absolute velocities. We must now look at this somewhat more in detail, especially in reference to *angular* as well as *linear* velocity.

When a body is turning about any fixed axis its motion is characterised by two conditions: (i.) the angular velocity of every point in it is equal, and (ii.) the linear velocities of its different points are proportional to their radial distances from the fixed axis, the linear velocities of points at equal distances from this axis being therefore equal. These conditions being characteristic of rotation simply, without reference to whether it occur for a short or a long time, are as entirely applicable to rotation about a virtual as about a permanent axis or centre. The difference is merely that in the former case the results obtained apply merely to one position of the body, while in the latter they apply equally to all its positions. We have seen that the motion of every link in a mechanism relatively to every other may at any instant be considered as a simple rotation about some point in that other. Hence it follows that at any instant every point in a link has the same angular velocity—that it describes, that is, equal angles in equal times.[1] It follows also that the linear velocities of different points in any link vary in direct proportion to the virtual radii of those points. Take Fig. 35 as an example, supposing *d* to be fixed, and the motions of the other three links observed relatively to it. Every point in *a* is, at the instant, turning with the same angular velocity about O_{ad}, every point in *b* with the same angular velocity about O_{bd}, and every point in *c* with the same angular velocity about O_{cd}. Further, a point in *a* at any given distance from O_{ad} moves with just half the linear velocity of a point in *a* twice as far from O_{ad}, and with double the linear velocity of a point half its distance from O_{ad}, and similarly with the other links, whether the centres about which they are turning be permanent or virtual.

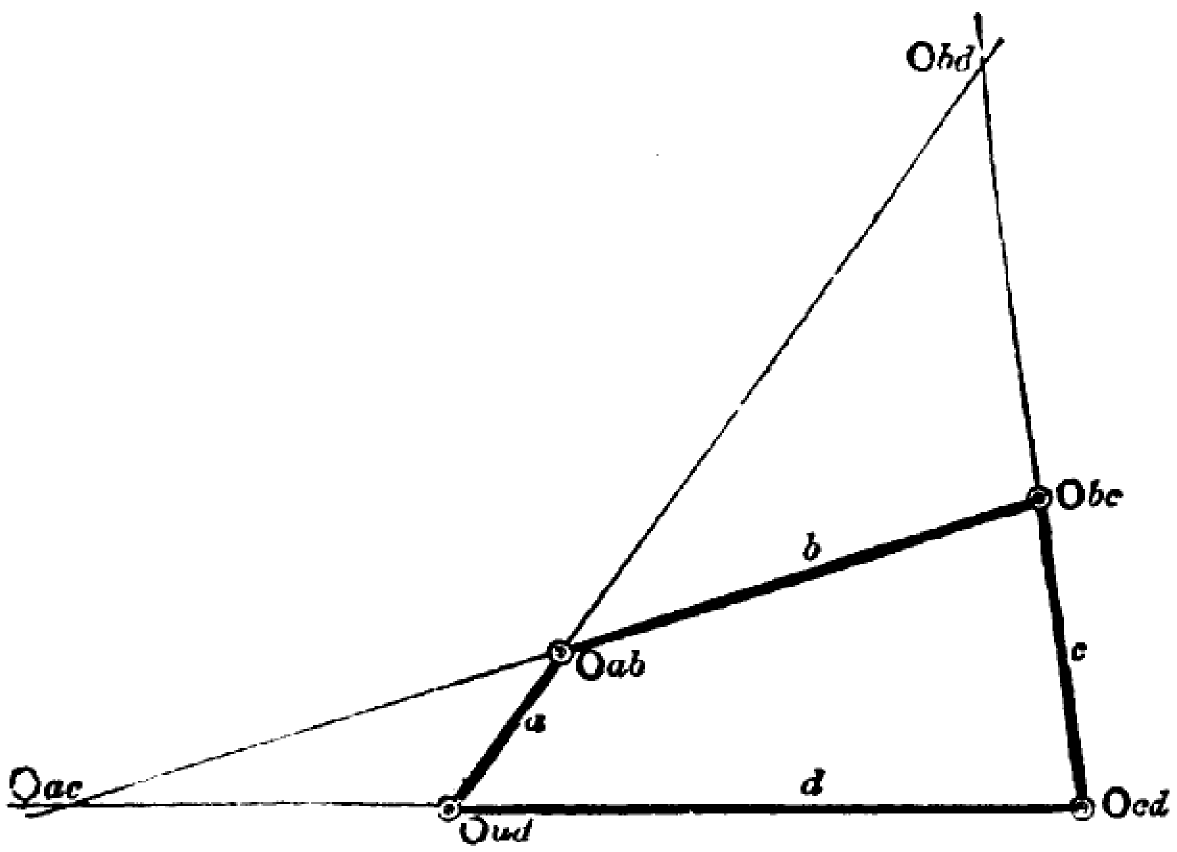

FIG. 35.

As we have seen, this makes it an extremely simple matter to find the velocity of all the points in any link if

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[1] More fully that all the points *would* describe equal angles in equal times if they continued to move with the velocities which they have at the instant of observation.

only that of one point be known. Suppose, for instance, that the velocity of the point A_1 (Fig. 36) be given, to find that of A_2, both points belonging to the link *a*. Arithmetically it might be found by measuring $\overline{O_{ad}A_2}$ and $\overline{O_{ad}A_1}$, to any scale, and multiplying the given velocity by the ratio between them, *i.e.* by $\frac{\text{virt. rad. } A_2}{\text{virt. rad. } A_1}$. We shall

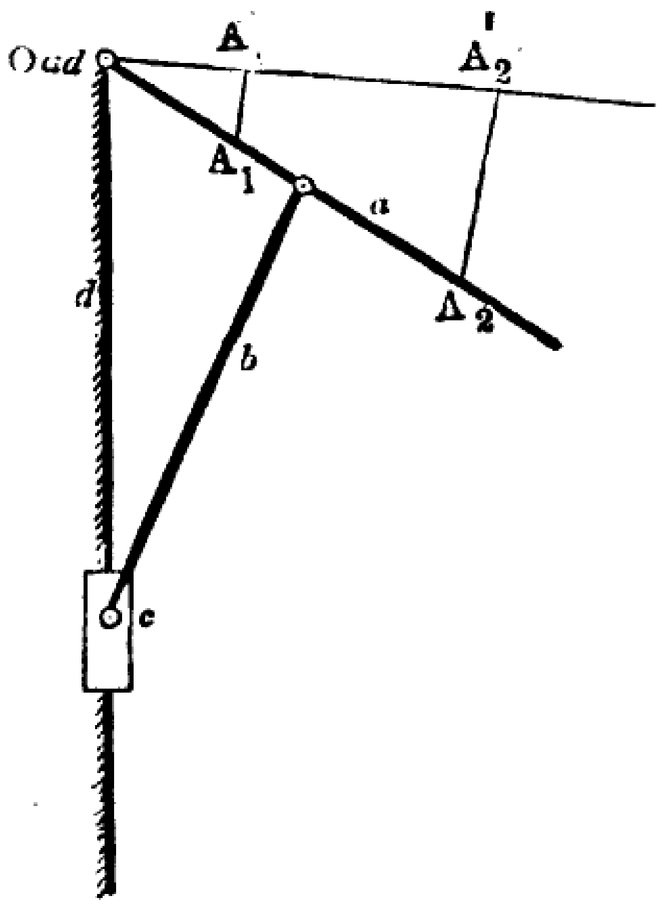

FIG. 36.

find it often more convenient, however, and it involves less measurement and no arithmetical multiplication, to solve the problem by a construction, as follows: Set off $A_1A'_1$ through the point A_1 in any convenient direction, to represent the given velocity of that point on any scale. Through O_{ad} draw a line through A'_1, and through A_2 a line parallel to $A_1A'_1$, calling the join of these lines A'_2;—the segment $A_2A'_2$ represents the velocity of A_2 on the same scale as that on which $A_1A'_1$ represents that of A_1. For the ratio

$$\frac{A_2A'_2}{A_1A'_1} = \frac{O_{ad}A_2}{O_{ad}A_1} = \frac{\text{virtual radius of } A_2}{\text{virtual radius of } A_1}.$$

Fig. 37 shows another construction, and one often more convenient than the foregoing, for solving a similar problem. Let $B_1 B_2$ be two points of a link b, and let $B_1 B'_1$ be the known velocity of B_1, to find that of B_2. Join both points to the virtual centre of b relatively to the fixed link, viz. O_{bd}.

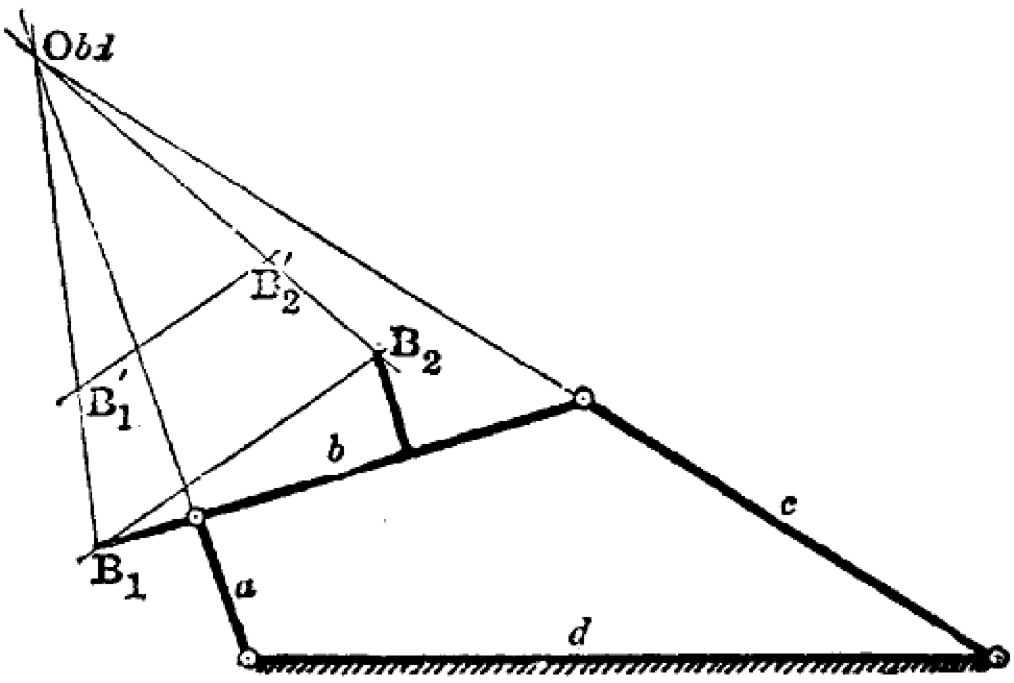

FIG. 37.

Join also B_1 and B_2, set off $B_1 B'_1$ along the radius of B_1 and draw $B'_1 B'_2$ parallel to $B_1 B_2$. Then $B_2 B'_2$ represents the linear velocity of the point B_2 on the same scale as that used in setting off $B_1 B'_1$. The proof is the same as before, simply that (by similarity of triangles)

$$\frac{B_1B'_1}{B_2B'_2} = \frac{O_{bd}B_1}{O_{bd}B_2} = \frac{\text{virtual radius } B_1}{\text{virtual radius } B_2}, \text{ as was required.}$$

It should be always most distinctly remembered that the bodies which are represented in our figures by straight links may be of any form whatever (see p. 67). We shall find that we have very often to do with points like B_2, Fig. 37, not lying at all on the axes of the bodies to which they belong. It should be noticed also that the line $A_1 A'_1$, &c., Fig. 36, were not set off in the direction of motion of A_1, &c., but in any direction that happened to be convenient.

We have compared the linear velocities of points of one and the same link,—but we can in just the same way compare the velocities of points in different links, or find the velocities of such points, if that of any one point be given. We do this by help of the theorem which we have already so often utilised, that the virtual centre of any link relatively to any other is a point common to both,—a point which has the same motion to whichever of the links we suppose it to belong. Let the velocity of a point A_1 on the link a, for instance, be given;—to find from it the velocity of a point B_1 on the link b. The process is simply to find first the velocity of the common point of a and b as a point of a, and then treating it as a point in b to find from it the velocity of B_1. The necessary construction is shown in Fig. 38. $A_1A'_1$ is drawn to scale in any convenient direction for the velocity of A_1; by the former construction $O_{ab}A$ represents on the same scale the velocity of O_{ab} considered as a point of a. But this point has the same velocity as a point of b, so that by joining A to O_{bd} and

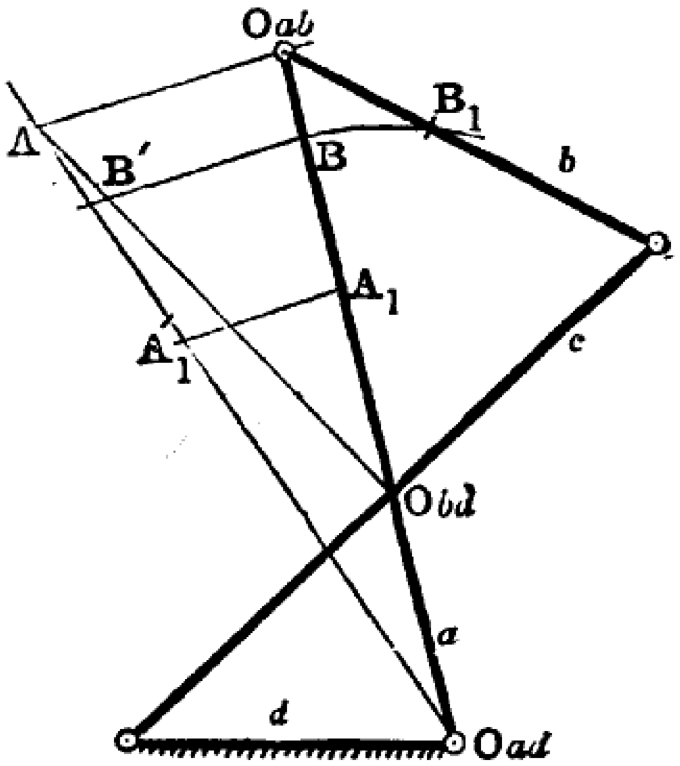

FIG. 38.

carrying the radius of B_1 round to B, as in the figure, we get BB' for the velocity of B_1, to be measured on the same scale as before.

The construction applies equally to opposite as to adjacent links. To find, for instance, the velocity of the point C_1 in c, having given the velocity of A_1 in a as before, we should proceed as in Fig. 39, finding the velocity of the point O_{ac} as a point in a, and then, treating it as a point of c, obtaining by its help the velocity of C_1. For convenience' sake we carry A_1 round to the line which is the axis of d, then setting off AA' as before, we obtain the line $O_{ac}A_2$ as the velocity of the point O_{ac}. Joining A_2 to O_{cd}, and carrying C_1 over to the axis of d (as we had previously done with A_1), we can at once draw CC' parallel to AA', and representing on the same scale the velocity of C_1.

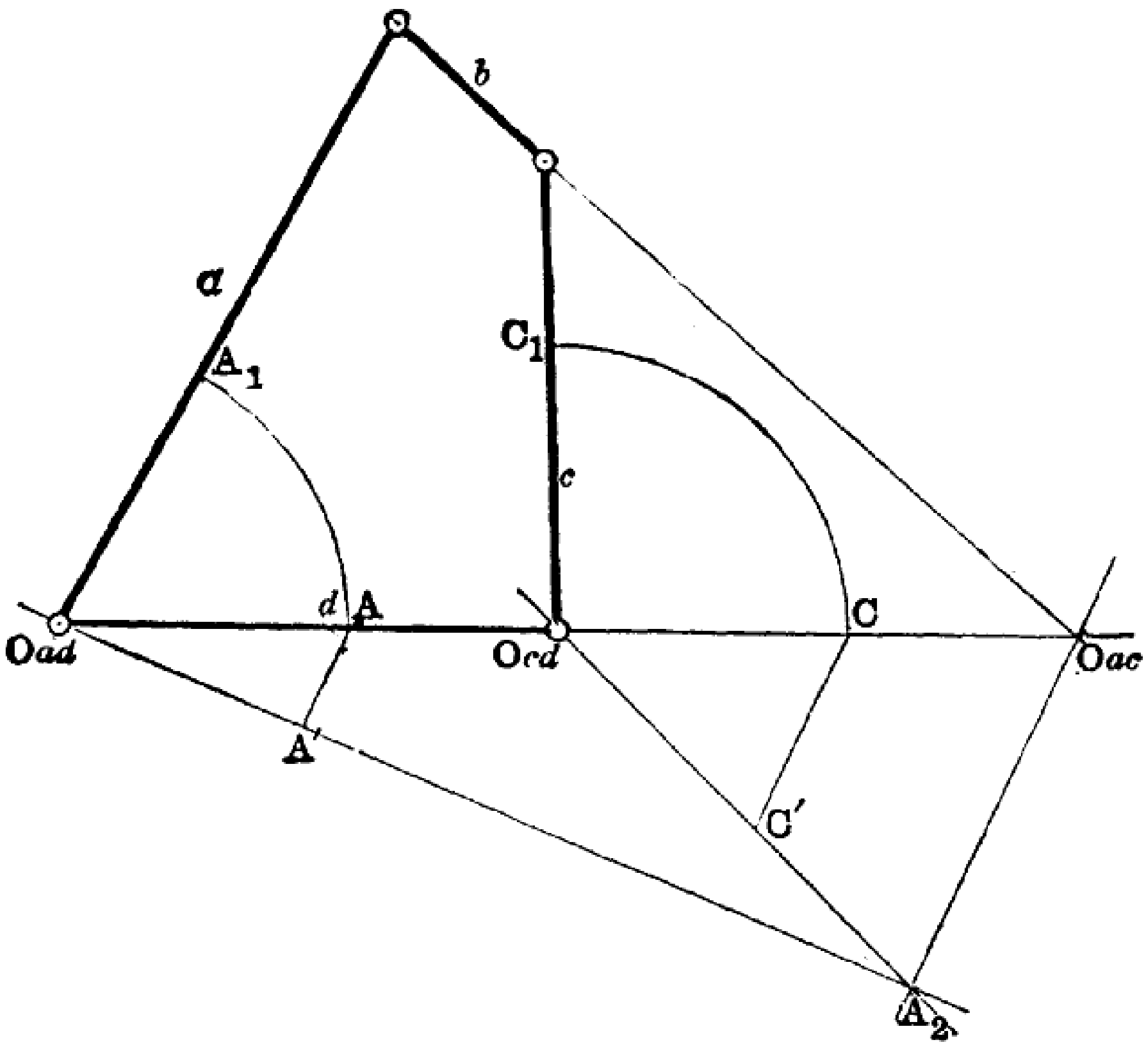

FIG. 39.