CHAPTER VIII.

STATIC EQUILIBRIUM.

§ 35.—CLASSIFICATION OF THE FORCES ACTING ON A MECHANISM.

ON every link in a mechanism, including, of course, the fixed link, there are usually a number of forces acting. We shall find it possible to classify the possible force conditions as simply as we have classified the possible conditions as to velocity and acceleration.

In the first place, the whole forces acting upon and between the links of any mechanism can be divided into two classes, between which it is quite easy to distinguish. Some of the forces, namely, are entirely external to the mechanism itself, and both in direction and magnitude may be independent of it; such forces are called **external forces.** The weight hanging from a crane, the resistance of a piece of iron to the edge of a cutting tool, the pressure of steam on a piston, are examples of such external forces. The weights of the individual links in any mechanism also fall into this category.

When any such external forces act at different points upon a mechanism—whether or not they cause the mechanism to move—they give rise to other forces acting from link to link of the mechanism, determined in magnitude by the external forces, but fixed in direction solely by the nature of the mechanism itself. These forces, in default of a better name, we shall call **pressures.** It would be misleading to call them "internal" as opposed to the "external" forces, for although they are internal in respect to the mechanism as a whole they are external to its links individually. In such an example as that of Fig. 103 the *external forces* are the two weights W_b and W_d, whose magnitudes and directions may be anything whatever[1] that we choose. Besides these there are in the mechanism forces

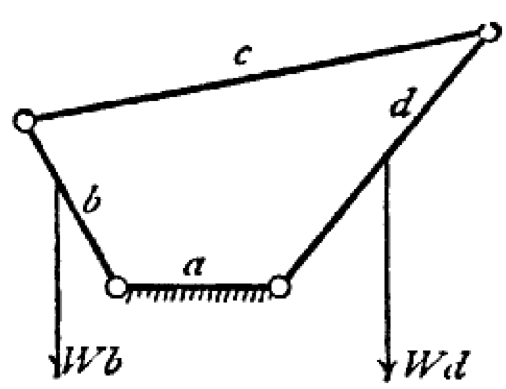

FIG. 103.

exerted by each link upon those next adjacent to it, whose total magnitudes are determined by the external forces, but whose directions and relative magnitudes are fixed by the mechanism itself. These are the *pressures.* The pressures exerted by *b* and *d* upon *c* are forces external to *c*, although not external to the mechanism. Similarly the pressures exerted by *a* and *c* upon *b* are external to it just as much as the external force W_b. But again they are not external to the mechanism, and therefore do not receive the name of external forces.

Pressures being by definition actions between adjacent links, occur always at the surfaces or lines of contact of the pairs of elements. We might say that they occurred always

[1] "Anything whatever," because we make no pre-supposition that the mechanism shall be "balanced" under them.

at the *joints* if we had only to do with turning pairs, but sliding surfaces are not generally included under the head of joints, although they are equally important to us as pairs. But it is clear that the pressures acting at different points, perhaps points very far apart, on the links, must be transmitted from point to point through the material of the links themselves. These transmitting, molecular forces might correctly be called internal forces; they have, however, received the more convenient name of **stresses**, which we shall always use to designate them.

In rigid bodies, stress may be defined as **resistance to alteration of form**; in fluids—which occasionally form part of machines, (see p. 3)—as **resistance to alteration of volume.** It is this capacity of the material of the links to exert stress in such fashion as to preserve their forms sensibly unaltered that justifies us in treating the virtual centre as a fixed point (p. 44). Could the shape of the links alter sensibly, the position of the virtual centre would be to a corresponding extent variable, the machine would become useless for its own proper purposes, and our method of examination would become inapplicable.

So long as the links of a mechanism are either stationary or moving with constant velocity, there come into question only these three—external forces, pressures, and stresses. Pressures act on each link from the next one at every pairing. The pressure of the link *a* on the link *b* is external to *b*, the pressure of *b* upon *a* at the same place is external to *a*, and so on. But no pressures can exist unless in the first instance they are called into existence by external forces acting on the mechanism. For the pressures may be taken to represent the resistance of the links (consequent on the manner in which they are connected together) to change of relative position, just as the stresses represent the resistance of the molecules (consequent on the manner in which *they* are connected) to change of relative position. The difference between the two cases is that the links are allowed to change their positions to a very large extent, and the molecules only to a very small one. The external forces may act on only one link of the whole mechanism, or on all, or on any number of the links. The mere weight of the links themselves may form a most important part of these forces, or may (as in a horizontal steam engine) be fairly negligible in comparison with the rest. The stresses, as representing entirely intermolecular action, may be left out of account here, it being presupposed only that the links are made of such material and dimensions as will keep the stresses in them so small that their change of form under pressure may be safely neglected. The stresses will then stand in the same general relation to the pressures that the pressures do to the external forces, except that wherever an external force acts on a link along with pressures it takes exactly the position of a pressure in causing stress.

The last paragraph has contained a general statement of the relations between stress, pressure, and external force in the case of bodies stationary or moving with constant velocity. When a body has acceleration, a force not falling properly under either of these three heads has to be taken into account. A body offers no resistance to continuance of motion in its own direction with its own velocity, but it cannot be accelerated without the action of force. This fact, which is Newton's "first law," is at the foundation of our whole study of dynamics. But it involves directly the converse fact that every body simply in virtue of its existence offers resistance to acceleration. This resistance is exactly measured by the force necessary to cause the acceleration, is equal and opposite to it, stands to it, in fact, in the relation of reaction to action. Neither can exist without the other; either may be looked at alone, but only if we do not forget that it is only half of a duality.[1]

When, therefore, any link of a mechanism is undergoing acceleration, its **resistance to acceleration**—a quantity proportional directly to its mass, as well as to the acceleration, but for which, unfortunately, we have no single word—is a force which has to be taken into account along with the rest, and which falls neither into the class of external forces nor into that of pressures, as we have defined them. We shall find presently that problems involving "resistance due to acceleration" are not more difficult to deal with than any others.

§ 36.—EQUILIBRIUM—STATIC AND KINETIC.

So long as the form of a body is not actually undergoing change—lengthening, shortening, distorting, etc.—the body is said to be in **equilibrium.** This equilibrium is called **static** if the body is stationary or moving with uniform velocity, and **kinetic** if it is undergoing acceleration.

For a body to be in static equilibrium it is necessary simply that the external forces acting upon it should not be such as could, in their united action, cause acceleration. Now the united or total action of any system of forces on a body is in every respect, except as to the stresses caused by the forces, the same as the action of the resultant or sum of that system of forces. The sum of any number of forces

[1] We talk similarly of the pressure of a girder on its abutment or of the reaction of the abutment against the girder. Neither can exist without the other, but without losing sight of the duality we often for simplicity's sake speak of only one.

may be either (i) zero, (ii) a single finite force of definite direction and position, or (iii) a couple, which has sense and has also magnitude measured as a moment, but has neither magnitude as a force nor any position or direction. So long as the forces are all in one plane, the condition always presupposed in this part of our work, no other condition than one of these three is possible.

If the sum of all the external forces be zero the body must be in static equilibrium, for zero force must cause zero acceleration.

If the sum of all the external forces acting on any link of a mechanism be a single force the equilibrium of the link is static or kinetic according to the position of that force. If the force passes through the virtual centre it can give the body no acceleration, because that point is a fixed one; no force whatever by acting on it can either make the body move or change its motion if it is already moving. In every other case a single force can and must cause the body to be accelerated. This may be summed up by saying that **if the sum of the external forces acting on any link of a mechanism be a single force, the link will be in static equilibrium only if that force act through its virtual centre relatively to the fixed link.**

If the sum of all the forces acting on any link of a mechanism be a couple, the condition of the link depends on the position of its virtual centre. If the link has a motion of translation only it will be in equilibrium, because its virtual centre is at infinity; in all other cases it must be undergoing acceleration. For looking at a couple merely as two equal, parallel, and opposite forces, there is no difficulty in seeing that it cannot cause acceleration in a body whose only possible motion is one of translation in one