CHAPTER IX.

PROBLEMS IN MACHINE DYNAMICS.

§ 44.—TRAIN RESISTANCE.

In the commencement of the last chapter it was pointed out that as long as a body was not actually changing its form it was said to be *in equilibrium.*

Further, we have seen that although any force, however small, inevitably alters the form of the bodies, however rigid, upon which it acts, yet that these alterations of form are, in a machine, intentionally made so minute that we are able to neglect them, and to treat the various bodies forming part of the machine as if their forms really remained unchanged.[1] We called the conditions of equilibrium **static** in the case where there were acting only external forces and pressures[2] in addition to the stresses in the links. This, we saw, corresponded to a condition either of rest or uniform velocity on the part of each body in the machine. We have examined in the last chapter most of the principal problems connected with the static equilibrium of bodies having constrained motion. In the present chapter we shall examine

[1] We neglect here the case of springs, leather belts, and the one or two other instances in which the alteration of form under external force is comparatively great.

[2] See § 36, p. 263.

a number of problems of a kind often distinguished from the former as dynamic instead of static, but which it is perhaps better to distinguish as problems of **kinetic**, instead of **static, equilibrium.** The difference between the two conditions is not the difference between no force and force, or between rest and motion, but between rest *or* motion with uniform velocity, and motion with varying velocity, that is, with acceleration. In this case not only external forces and pressures act upon the links, but also resistances due to their acceleration, which resistances may be positive or negative according to the sense of the acceleration. These resistances are forces whose magnitudes depend upon the *masses* of the links themselves, and are in this respect essentially different from the forces and pressures previously considered, in working with which the masses of the bodies acted upon never entered into the question. It is on this account that we separate the problems in which they occur from those formerly considered, and not because both sets of problems do not alike relate to conditions of equilibrium.

In the present chapter we shall examine in detail certain typical and very important problems of kinetic equilibrium representing those which have actually to be solved in engineering work. We shall take first in this section, as perhaps the simplest case that can be taken, the case of the motion of a train, from which we have already used several illustrations in § 28. We shall next examine dynamically a simple direct-acting pumping engine, then a Cornish engine, then the driving mechanism of an ordinary horizontal steam engine, next the fly-wheel of such an engine, and its connecting rod, and lastly the case of centrifugal governors.

Let us suppose that a train weighing 80 tons, or say 180,000 pounds, is running on a level at 40 miles per hour, and that 40 wheels are simultaneously braked, with such a pressure on each brake as gives a frictional resistance of 30 pounds at the periphery of each wheel. The total brake pressure is thus $40 \times 30 = 1200$ pounds. The speed of the train is 58·4 feet per second. It is required to draw a diagram of the stop of the train.

It may be stated at once that in such a very simple case as this one, diagrams are by no means necessary, nor is a graphic solution to be preferred to ordinary calculation if only final results are required. Both calculation and diagram will be given here, the latter partly for the sake of the determination of its scales, and partly because by its nature it gives not only a final result—the distance run before stopping, or whatever it may be—but also a pictorial representation of the whole process of stopping, which it is in many cases important to follow, and which otherwise can only be understood by a series of separate calculations, together much more trouble than the drawing of the diagram.

The distance which will be run before stopping can be found at once by calculating the kinetic energy stored up in the train, as it moves with the given velocity, and dividing this quantity by the resistance to its forward motion, namely, the resistance of the train proper plus the added artificial resistance of the brakes. The stored-up energy is

$$\frac{180{,}000 \times 58{\cdot}4^2}{64{\cdot}4} = 9{,}530{,}000 \text{ foot-pounds.}$$

The normal resistance of the train on a level may be taken as 8 pounds per ton, or 640 pounds. The brake resistance is 1,200 pounds, but it has to be overcome through a distance π times as great as that moved through by the train as a whole. The total resistance is therefore $640 + (\pi \times 1200)$ = (say) 4,400 pounds. The distance which the train will run before stopping is therefore $\frac{9,530,000}{4,400} = 2,170$ feet, or about two-fifths of a mile.

The diagram (Fig. 148) which represents this case is drawn with the following scales :—

(1) Force or pressure scale 2,000 pounds = 1 inch.
(2) Distance scale 1,200 feet = 1 inch.
(3) Velocity scale 20·6 feet per second = 1 inch.
(4) Acceleration scale 0·356 foot-seconds per second = 1 inch.

The first two scales are taken arbitrarily as may be convenient. The acceleration scale is derived from the force scale by the relation $a = f\frac{g}{w}$ (p. 222), which tells us that 1 inch must stand for $\frac{g}{w}$ times as many units of acceleration as of pressure. This fraction is here ·000178, so that the acceleration scale is 2000 times this, as given above. The velocity scale is derived from the distance and acceleration scales also as before, one inch standing for $\sqrt{0·356 \times 1200}$, or 20·6 feet per second (very nearly).

The distance OA is first set up for the initial velocity of the train, and ON for the acceleration (here negative, of course). AB_1 is drawn at right angles to NA. BB_1 is a vertical at any convenient distance from the origin. BN_1 is the acceleration (= ON), and the next segment of the velocity curve, B_1C_1, is drawn at right angles to N_1B_1 and so on. This gives the curve $AB_1C_1 \ldots S_1$, and the distance OS_1 run before the train stops (see p. 204). But the process is obviously one which gives cumulative errors, and these are too great to be neglected. For not only is the point C_1, for instance, too high in position on account of the substi-

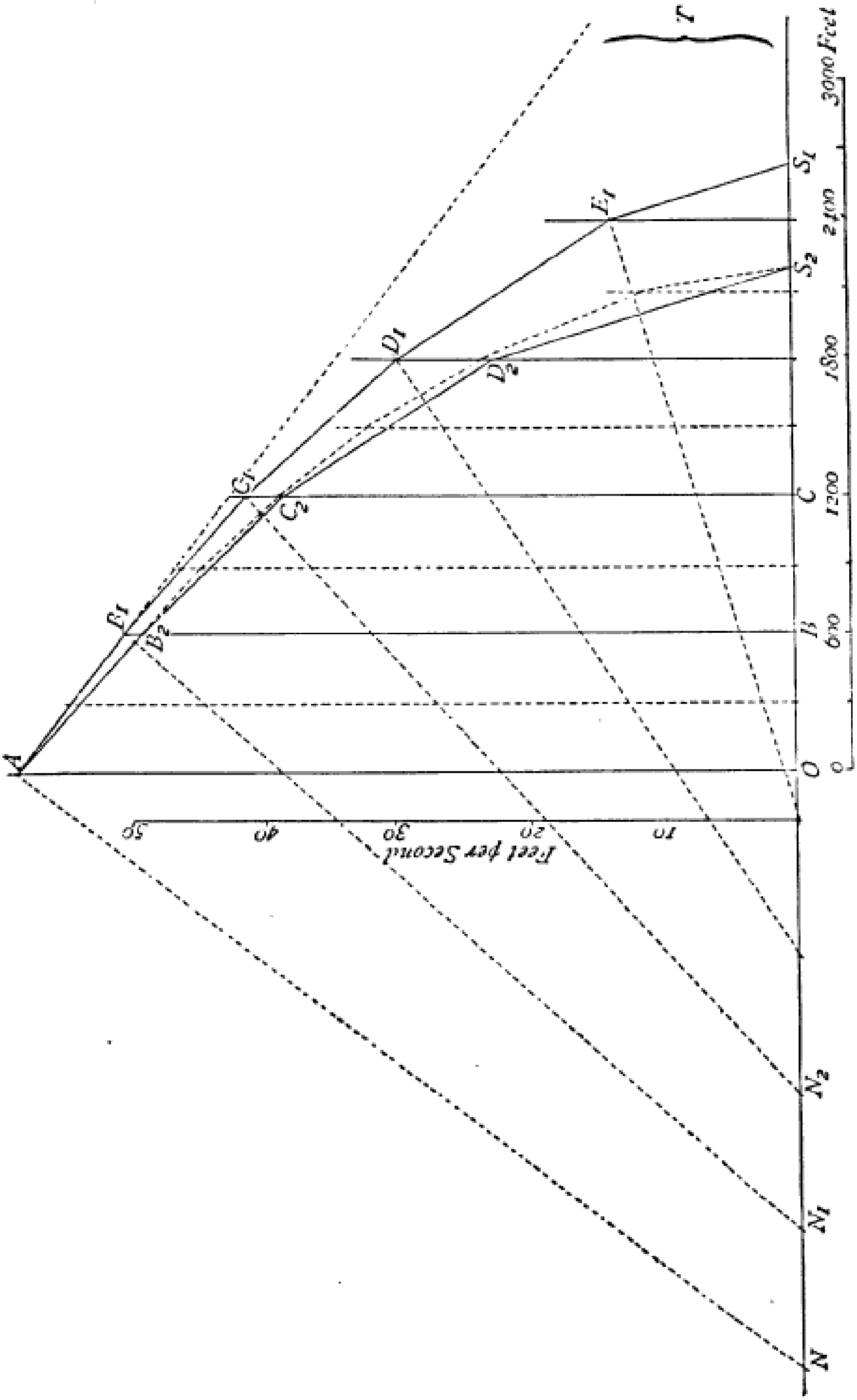
A
B1
B2
C1
C2
D1
D2
E1
S1
S2
T
O
B
C
N
N1
N2
Feet per Second
10
20
30
40
50
0
600
1200
1800
2400
3000 Feet

FIG. 148.

tution of the straight line B_1C_1 for an arc convex upwards, but it is further misplaced by the similar error in the position of B_1, which makes the angle BB_1C_1 too great. The error of each point in the curve is thus greater than that of the preceding one, and the whole distance OS_1 is (in this example) nearly 20 per cent. too great. The longer the distance intervals OB, BC, &c. be taken the greater does this error become. For any reasonable distance intervals, however, the point S_1 can be obtained very approximately by drawing a second polygon $AB_2C_2 \ldots S_2$, so that AB_2 is parallel to B_1C_1, B_2C_2 parallel to C_1D_1, C_2D_2 to D_1E_1 and so on. This construction makes the distance OS_2 in the figure very slightly too small. But although this error of S_2 is here quite negligible, yet with the distance intervals shown the intermediate points B_2C_2 and D_2 are very sensibly out of position. By taking the distance intervals sufficiently small, these points also can be brought sensibly right. In Fig. 148 the dotted curve is drawn like $AB_2C_2S_2$, but for distance intervals $= \frac{OB}{2}$ or 300 feet, and its points sensibly coincide with calculated points.

As the acceleration, which is the sub-normal to the velocity curve (p. 205), is constant, we know that the curve itself is a parabola, and as such it can easily be drawn. The point S_2 must lie midway between O and T, where AB_1 cuts the axis. In starting such a diagram the two quantities to be set out are the velocity and the acceleration. It is not really necessary, however, to set out the latter, for we know that it is proportional to the force, hence ON is really set out equal to the force (here resistance) causing (negative) acceleration. Knowledge of the scale on which ON represents the acceleration is only required in order to find on what scale the velocity must be drawn in order to correspond to our assumed distance scale, or—if the velocity scale has been arbitrarily assumed—to determine on what scale OD must be read of as distance.

Fig. 149 shows the same problem worked out on a time, instead of a distance, base. The velocity curve is here, as

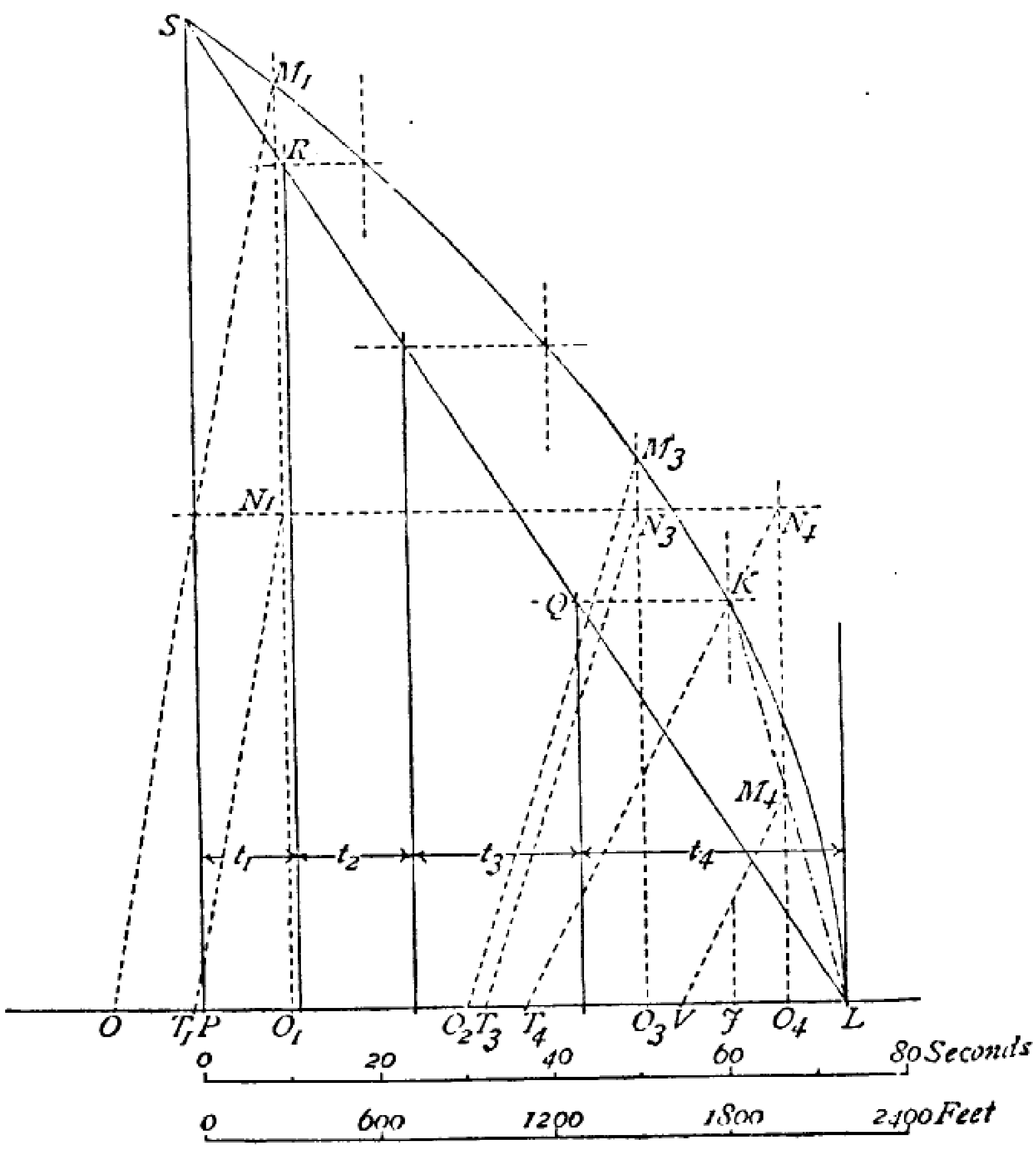

FIG. 149.

in Fig. 92, a straight line, the acceleration being constant. The acceleration is equal to $\frac{f}{m}$ or $f\,\frac{g}{w}$, which is here $4400 \times \frac{32 \cdot 2}{180{,}000} = 0 \cdot 787$ foot-seconds per second. The