CHAPTER XII.

FRICTION IN MECHANISMS AND MACHINES.

§ 71.—FRICTION.

WHEN two surfaces are pressed together it is found that one cannot be moved along and relatively to the other, without the exertion of some definite effort. The resistance, to balance which this effort has to be exerted, is called the **friction** between the surfaces. It can be measured as a force acting from one surface to the other in the direction

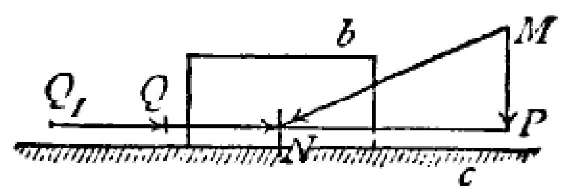

FIG. 326.

of their relative motion, and with a sense such as to offer resistance to that motion. On this account it is sometimes loosely spoken of, without sufficient qualification, as a force which tends always to oppose and never to produce motion.[1] Let *b* and *c* (Fig. 326) be two bodies touching one another, and let *b* slide upon *c* (supposed fixed) under the action of the

[1] A proposition somewhat fiercely attacked by Reuleaux, *Kinematics of Machinery*, p. 594.

force MN. This force has a component MP pressing the surfaces together and causing friction, and a component PN in the direction of motion. Let the external resistance to the motion of b, independently of friction, be QN. Then, disregarding friction, the body b will be receiving an acceleration of $\frac{PN-QN}{m}$ foot-seconds per second, its mass being supposed to be m. If now Q_1Q be the frictional resistance produced by the pressure component MP, the body will be receiving an acceleration only of $\frac{PN-(QN+Q_1Q)}{m}$ foot-seconds per second, and the difference between these two values is the acceleration caused by the frictional resistance. If $Q_1N=PN$ the body b will be moving with a uniform velocity, just as it would do if the friction were absent and QN were equal to PN. We do not say in such a case that the resistance QN does not produce motion; we treat it, on the contrary, as a force in every respect similar to the effort PN, but differing from it in sense. There seems no really sufficient reason for treating frictional resistances in any different manner.

A frictional resistance has always a sense opposite to that of the relative motion of the bodies between which it acts. So far, therefore, as the motion of these bodies relatively to each other is concerned, the acceleration produced by it is always negative. But it is often utilised in order to produce positive acceleration of one of the bodies relatively to a third. Fig. 327 illustrates this, where c is not itself a fixed body, but one capable of sliding upon a fixed body d. Suppose that b were fixed to c by bolts whose united resistance to fracture was RN. Then if the effort could exceed this value the bolts would shear and b would move upon c. But so long as the effort PN is less than RN, the two bodies could not move relatively to each other, and *PN* would be balanced by a stress in the bolts (that is, by a *portion* of *RN*), exactly equal to itself, as *QN*. The stress in the bolts is what may be called a **derived force**, which has a maximum value *RN*, but whose actual value is any magnitude less than this which is necessary to balance the external force opposed to it, here *PN*. This precisely represents the conditions of the case if we substitute frictional resistance between the surfaces for the shearing resistance of the supposed bolts. The friction is a derived force depending here upon the pressure *MP*, and upon the state of the surfaces. It has some maximum value (which we may suppose to be *RN*) entirely independent of *PN*. If *PN* exceed that value the bodies

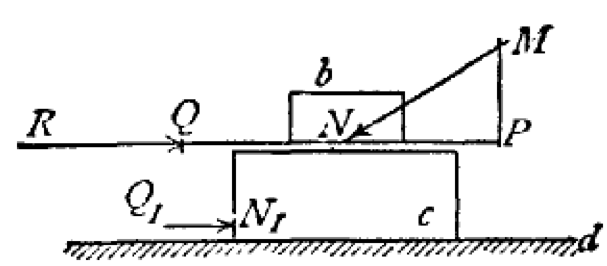

FIG. 327.

will move relatively to each other as in Fig. 326. If *PN* falls short of that value it will be balanced by a frictional resistance exactly equal to itself, the rest of the possible frictional resistance being non-existent in the same sense as the balance of possible stress in the bolts. Hence if *RN* be in this case the maximum possible friction, and *PN* the only driving effort, *b* will remain stationary relatively to *c*, under the equal and opposite forces *PN* and *QN*. But if *c* can move relatively to *d* under some sufficiently small resistance (frictional or other), Q_1N_1, it will be set in motion, receiving the acceleration $\frac{PN - Q_1N_1}{m}$ foot-seconds per second, just as before. In this case it seems legitimate to say that friction is the cause of *positive* acceleration and not negative, for it is the friction between *b* and *c* which transfers the driving effort *PN* from *b* to *c*, and which in that sense is the cause of the positive acceleration of *c* relatively to *d*.

Both the cases described occur continually in machinery. Wherever two surfaces have to be rubbed together (as in every pin joint or other pair of elements throughout the whole machine), frictional resistances cause negative acceleration, work is expended in overcoming them, they diminish the efficiency of the machine, and it is the object of the engineer to reduce them to the furthest possible extent. In all belt and rope gearing, however (and in a few other cases), the frictional resistance between two bodies (the belt and the pulley) is utilised as the sole means of giving motion to one of them (relatively to a third) and transmitting work to it. It is essential for this purpose that the possible friction between the two bodies (as *b* and *c* in Fig. 327) should be as *great* as possible, and its magnitude only affects the efficiency of the machine if it is too small, so as to permit the relative motion of the bodies which it is intended to prevent.

Experiments made upon the friction of bodies caused to slide upon one another without any, or with little, lubrication, at very moderate velocities, and with small intensities of pressure,[1] have established the facts that under these conditions the friction is independent of the area of contact and intensity of pressure, and is practically independent of the velocity of rubbing, being for any given pair of surfaces proportional simply to the total normal pressure. Under such conditions, therefore, the frictional resistance can be found at once for any known value of the pressure P by multiplying it by some co-efficient μ dependent essentially on the

[1] By *intensity* of pressure is meant, as formerly, pressure per unit of area.

nature of the surfaces, so that the value of the friction is written

$$F = \mu P.$$

The multiplier μ is called the **co-efficient of friction**, and is assumed to be fairly constant for given materials with such surfaces as are commonly used.

Engineers, however, have seldom to do with unlubricated rubbing surfaces, and they have to deal with surfaces moving often with very high velocities, and under very great and frequently varying pressures. Under these conditions the "laws" of friction, as they have just been stated, not only do not hold exactly true, but fail even to represent approximately the more complex phenomena with which they have to deal. At many speeds and loads which are of daily occurrence in machinery, velocity and intensity of pressure have an enormous effect on the friction, and not only these, but the temperature of the surfaces and the nature of the lubricant. The nature of the rubbing contact also, whether continuously in one sense or continually reversed, whether the surfaces be flat as in a guide, or cylindrical as in a bearing, whether contact exist throughout a surface or only along a line, very greatly affects the friction. The actual material of which the surfaces consist forms only one out of an immense number of conditions which determine friction under a given load. In fact, although all the rubbing surfaces in a machine were made of the same material, and had as nearly as possible the same smoothness, the co-efficient of friction, that is the quantity by which the total pressure on each surface would have to be multiplied to find the friction, instead of being practically constant, might be ten times [1] as great for some pairs of surfaces as for others. In

[1] Often enormously more than ten times. The particular number ten is not intended to have any special significance.

each particular pair of surfaces, with its own special area, velocity, form, amount of lubrication, and so on, the frictional resistance bears a different proportion to the load, and can be estimated from it only by the use of a different co-efficient. Under these circumstances it is perhaps misleading to retain the much-used phrase, "co-efficient of friction," for this inconstant multiplier. The co-efficient of friction has the certain definite meaning which has already been explained, and which limits its use to solid friction under certain simple conditions. It is so thoroughly associated with the idea that friction is proportional to load, that it seems unadvisable to call by its name a mere multiplier which may even itself vary inversely as the load. We shall, therefore, speak rather, in the following sections, of the **friction-factor** for a given pair of surfaces, meaning by this expression simply the ratio, dependent on all the varying conditions already mentioned, of the frictional resistance of those surfaces to the pressure causing it. We may, therefore, write $\frac{F}{P}=f$, the friction-factor, so that we still have

$$F=fP$$

but with the condition that f is a quantity whose value has to be separately considered for each set of conditions.

In every mechanical combination, from a pair of elements to a machine, some effort is at each instant expended in balancing friction, some work therefore is done, as the machine moves, merely in overcoming frictional resistance. If we call the remaining effort or work, as the case may be, the *nett* or *useful* effort or work of the combination, the ratio $\frac{\text{useful effort}}{\text{total effort}}$ or $\frac{\text{useful work}}{\text{total work}}$ is called the **efficiency** of the apparatus. Where the ratio is between amounts