PART I.

GEOMETRY OF MACHINERY.

CHAPTER I.

ELEMENTARY RULES IN DESCRIPTIVE GEOMETRY.

SECTION I.—*General Explanations—Projection of Points and Lines.*

6. **Descriptive Geometry** is the art of representing solid figures upon a plane surface. In the present chapter are given some general elementary rules in that art, whose application is of very frequent occurrence in designing mechanism. The more special and complex rules will be given in the ensuing chapters, in treating of the particular kinds of mechanism to which those rules belong.*

7. By the **Projection of a Point** upon a given plane is meant the foot of a perpendicular let fall from the point on the plane. For example, in fig. 1, X Z Z X represents a plane (called a plane of projection), A a point, and A B a perpendicular let fall from the point on the plane; the foot, B, of that perpendicular is the projection of the point A on the plane X Z Z X.

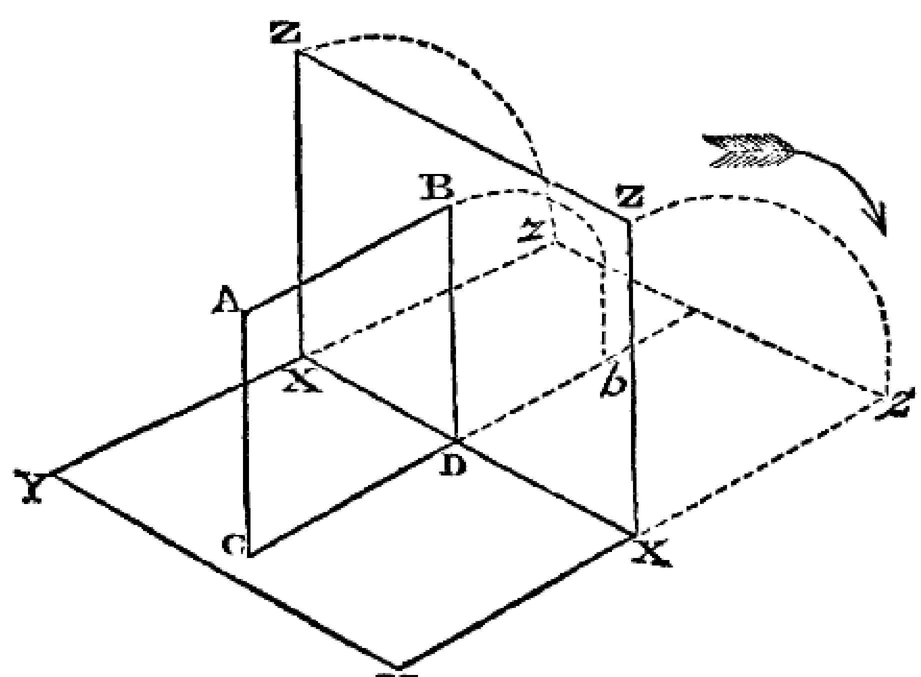

Fig. 1.

8. The **Position of a Point** is completely determined when its projections upon two planes not parallel to each other are known. In descriptive geometry a pair of planes of projection at right angles to each other are used; and in general one of these is vertical and the other horizontal. Thus, in fig. 1,

* For complete information on the subject of descriptive geometry, reference may be made to the works of Monge and Hachette in French, and of Dr. Woolley in English.

X Z Z X is the vertical plane of projection, and X Y Y X the horizontal plane of projection; B is the vertical projection, and C the horizontal projection of the point A; and those two projections completely determine the position of the point A; for no other point can have the same pair of projections.

9. The **Axis of Projection** is the line X X, in which the two planes of projection cut each other.

10. **Rabatment.**—When the two projections of an object are shown in one drawing, it is convenient to represent to the mind that the following process has been performed:—Suppose that the vertical plane of projection is hinged to the horizontal plane at the axis X X, and that after the projection of the object on the vertical plane has been made, that plane is turned about that axis until it lies flat in the position X *z z* X, so as to be continuous with the horizontal plane: thus bringing down the projection B to *b*. This process is called the *rabatment* of the vertical plane upon the horizontal plane (to use a term borrowed from the French "*rabattement*" by Dr. Woolley). The two points C and *b* are in one straight line perpendicular to X X. The process of rabatment may be conceived also to be performed upon a plane in any position when a figure contained in that plane is shown in its true dimensions on one of the planes of projection.

11. **Projections of Lines.**—The projection of a line is a line containing the projections of all the points of the projected line. The projection of a straight line perpendicular to the plane of projection is a point; for example, the projection on the vertical plane, X Z Z X (fig. 1), of the straight line A B, perpendicular to that plane, is the point B. The projection of a straight line in any other position relatively to the plane of projection is a straight line. If the projected line is parallel to the plane of projection, its projection is parallel and equal to the projected line itself; thus the projection on the horizontal plane, X Y Y X, of the horizontal straight line A B, is the parallel and equal line C D. If the projected line is oblique to the plane of projection, the projection is shorter than the original line.

The projections, on the same plane, of parallel and equal straight lines are parallel and equal. The projections, on the same plane, of parallel lines bearing given proportions to each other are parallel lines bearing the same proportions to each other. When the plane of a plane curved line is perpendicular to a plane of projection, the projection of the curve on this plane is a straight line, being the intersection of the plane of the curve with the plane of projection. When the plane of the projected curve is parallel to a plane of projection, the projection of the curve on this plane is similar and equal to the original curve. In all other cases, it follows from the preservation of the proportions of a set of parallel ordinates amongst their projections, that the projections of a plane curve of a given algebraical order are curves of the same algebraical order. The projections of a circle are ellipses; the projections of a parabola of a given order are parabolas of the same order. The projections of a straight tangent to a plane curve are straight tangents to the projections of that curve. The projections of a point of contrary flexure in a plane curve are points of contrary flexure in its projections.

12. **Drawings of a Machine.**—A third plane of projection, perpendicular to the first two, is often employed, not as being mathematically necessary, but as being more convenient for the representation of certain lines. Thus, for example, the drawings of a machine usually consist of three projections on three planes at right angles to each other; one horizontal (*the plan*), and the other two vertical (*the elevations*). Any two of those projections are mathematically sufficient to show the whole dimensions and figure of the machine; and from any two the third can be constructed; but it is convenient, for purposes of measurement, calculation, and construction, to have the whole three projections.

In the application of the rules about to be stated in the sequel of this Section, the two planes of projection may be held to represent any two of the three views of a machine; and the axis of projection will then have the directions stated in the following table:—

Views Represented by the Planes of Projection.	Direction of the Axis of Projection.
Longitudinal Elevation and Plan,...........	Longitudinal.
Longitudinal and Transverse Elevations,....	Vertical.
Plan and Transverse Elevation,..............	Transverse.

Projections of figures upon planes oblique to the principal planes of projection may be used for special purposes.

Section II.—*Traces of Lines and Surfaces.*

13. By a **Trace** is meant the intersection of a line with a surface, or of one surface with another. The trace of a line upon a surface is a point; the trace of one surface upon another is a line.

In descriptive geometry the term *traces* is specially employed, when not otherwise specified, to denote the intersections of a line or surface with the planes of projection.

14. **Traces of a Straight Line.**—The position of a *straight line* is completely determined when its traces are known. For example, the straight line A C, in fig. 2, has its position completely determined by its traces, A and C, being the points where it cuts the

two planes of projection. The *rabatment* of the trace C is represented by *c*.

A straight line parallel to one of the planes of projection has only one trace, being the point where it cuts the other plane of projection.

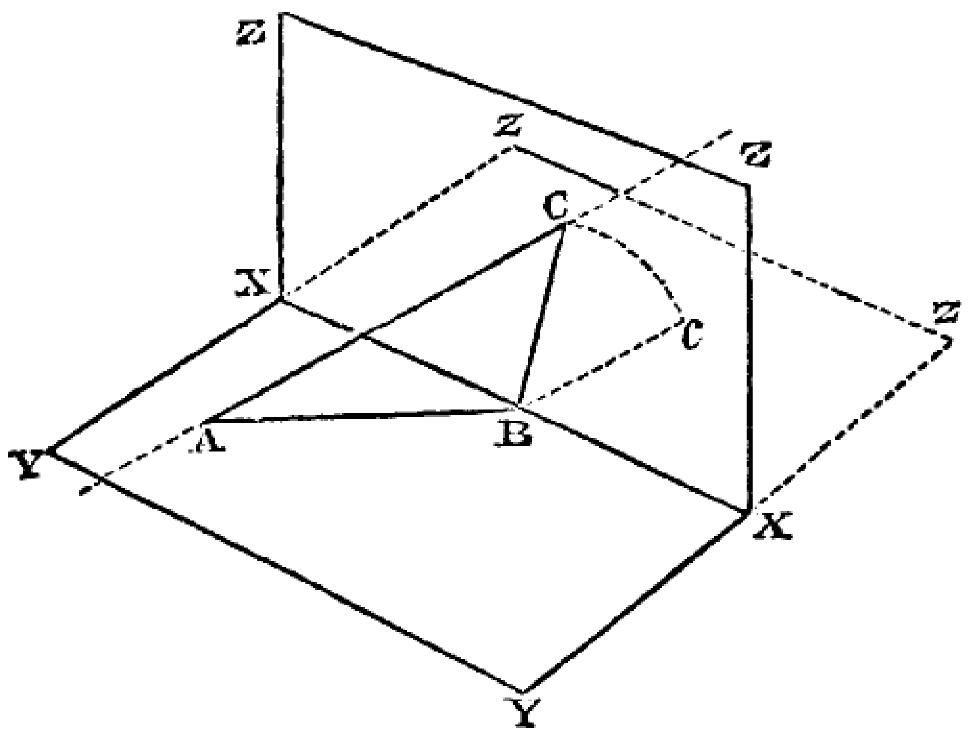

Fig. 2.

A straight line parallel to the axis of projection has no traces.

15. The **Traces of a Plane** are straight lines which (unless they are both parallel to the axis of projection) meet that axis in one point. The position of a plane is completely determined when its traces are known. For example, the plane A B C, in fig. 2, has its position completely determined by its traces, B A and B C.

A plane perpendicular to one of the planes of projection has its trace on the other plane of projection perpendicular to the axis of projection. A plane perpendicular to both planes of projection has for its traces two lines perpendicular to the axis. Thus, in fig. 1, page 3, the traces of the plane A B C D are D C and D B, both perpendicular to X X.

A plane parallel to one of the planes of projection has a trace on the other plane of projection only, being a straight line parallel to X X.

If a plane traverses a straight line, the traces of the plane traverse the traces of the line.

SECTION III.—*Rules Relating to Straight Lines.*

16. **General Explanations.**—In each of the figures illustrating the following rules the axis of projection is represented by X X; and in general the part of the figure above that line represents the rabatment of the vertical plane of projection, and the part below, the horizontal plane of projection. The projections of points on the horizontal plane are in general marked with capital letters, and the projections on the vertical plane with small letters.

17. **Given** (in fig. 3), **the Traces,** A, *b*, **of a Straight Line, to Draw its Projections.**—From A and *b* let fall A *a* and *b* B perpendicular to X X. Then *a* will be the vertical projection of the

trace A, and B the horizontal projection of the trace *b*. Join *a b*, A B; these will be the projections required.

(It may here be remarked, that *a* A and *a b* are the traces of a plane traversing the given line, and perpendicular to the vertical plane of projection; and that B A and B *b* are the traces of a plane traversing the given line, and perpendicular to the horizontal plane of projection.)

18. **Given** (in fig. 3), **the Projections,** A B, *a b*, **of a Straight Line, to Find its Traces.**—From *a* and B, where the given projections meet the axis, draw *a* A and B *b* perpendicular to X X, cutting the given projections in A and *b* respectively. These points will be the required traces.

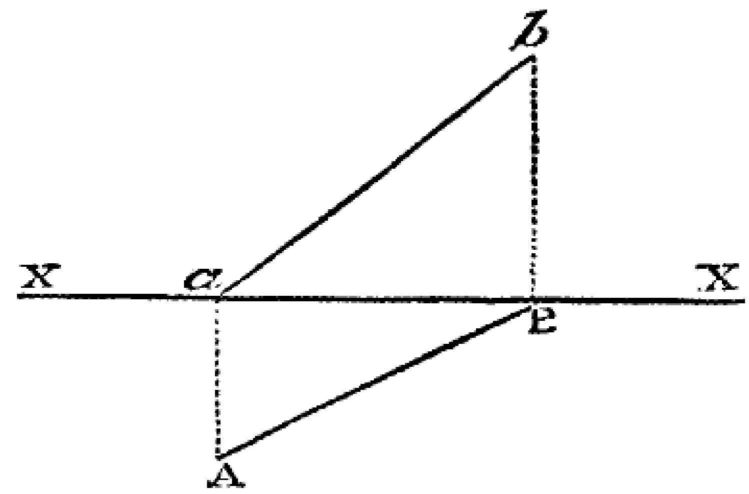

Fig. 3.

19. **Given, the Projections of two Points,** A, *a*, B, *b* (fig. 4), **to Measure the Distance between them.**—Join *a b*, A B; these will be the projections of the straight line to be measured. Through either end of either of those projections (as *b*) draw *d b e* parallel to X X; through the other end, *a*, of the same projection, draw *a d* perpendicular to X X, cutting *d b e* in *d*; make *d e* = the other projection, A B; join *a e*; this will be the length required.

The same operation may be performed on the other plane of projection.

20. **Given** (in fig. 4), **the Projections,** A, *a*, **of a Point, and the Projections,** A B, *a b*, **of a Straight Line through that Point, to Lay off a given Distance from the Point along the Line.** — In any convenient position, draw a straight line, B *b*, perpendicular to X X, meeting the projections of the given straight line in two points, B, *b*, which are the projections of one point; then perform the construction described in Article 19, so as to find *a e*. From the point *a*, in the line *a e*, lay off the given distance, *a f*. Through *f* draw *f h* parallel to X X, cutting *a b* in *g*; *a g* will be one of the projections of the given distance. Then draw *g* G perpen-

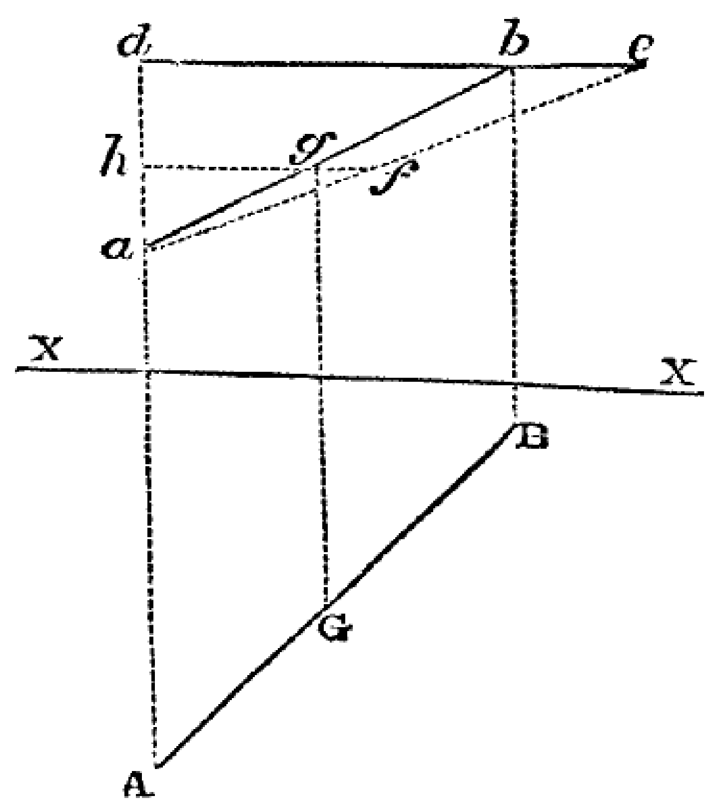

Fig. 4.