CHAPTER III.

OF THE MOTIONS OF SECONDARY MOVING PIECES.

67. **General Principles.** (*A. M.*, 383, 384, 503.)—In the present chapter the general principles only of the motions of secondary moving pieces in machines can be given, many of their most important applications being reserved for that chapter which will treat of "Aggregate Combinations in Mechanism," and some for the chapter on "Elementary Combinations." The mechanism for producing the motions of secondary moving pieces belongs wholly to those later chapters.

Secondary moving pieces have already been defined (in Article 37, page 17) as those which are carried by other moving pieces, or which have their motions not wholly guided by their connection with the frame. Their motions, therefore, are not restricted, like those of primary pieces, to translation in a straight line, rotation about a fixed axis, and that combination of those two motions which constitutes the motion of a screw with a fixed axis; they comprehend translations along curved lines of various figures, rotations about shifting axes, and various combinations of translations and rotations. The paths of points, too, in secondary pieces are not restricted to three forms—the straight line, the circle, and the helix; they comprehend a great variety of curved lines, both plane and of double curvature. The comparative motions of any two points in a primary piece are constant. The comparative motions of two points in a secondary piece very often vary from instant to instant as the piece changes its position.

In many cases the motions of secondary pieces are partially guided or restricted. For example, a secondary piece may be so guided that all its movements take place parallel to a fixed plane; in which case its motions are restricted to translations parallel to the fixed plane, and rotations about axes perpendicular to it; and the paths of its points are restricted to lines, straight or curved, in or parallel to that plane; and this restricted case is by far the most common in mechanism. Another kind of restriction on the movements of a secondary piece is when it turns about a ball and socket joint, or some equivalent contrivance, so that one point at the centre of the joint is kept fixed: in this case its motions are restricted to rotations about axes traversing that fixed point; and the motions of points in it are restricted to curves situated in spherical surfaces described about the fixed point. Cases in which the movements of secondary moving pieces are not restricted in one or other of those ways are comparatively rare.

The geometrical problems relating to the motions of secondary moving pieces may be divided into the two following classes:—

I. When the motions, in most cases, of two, and at furthest of three, points in a secondary moving piece are given, and it is required to find the motion of any other point in the piece, or of the piece as a whole. All problems of this class depend for their solution on the principle of Article 54, page 32.

II. When there are two moving pieces or moving points, C and B, the frame of the machine being denoted by A, and two out of the three motions of A, B, and C relatively to each other being given, it is required to find the third of those motions. All problems of this class depend for their solution on the principle (already stated in Article 42, page 21) that the motion of C relatively to A is the resultant of the motions of B relatively to A, and of C relatively to B.

68. **Translation of Secondary Moving Pieces.** (*A. M.*, 369.)—If, in a moving piece whose movements are not restricted, the directions of motion of three points not in the same straight line are parallel to each other and oblique to the plane of the three points; or if, in a moving piece restricted to movements parallel to one plane, the motions of two points are parallel to each other and oblique to the line of connection of the points; then the motion of the whole piece is a translation. All the points in the piece describe equal and similar paths, straight or curved; and all, at a given instant, move with equal velocities in parallel directions. The motion of any pair of points in the moving piece relatively to each other is nothing; and their comparative motion consists in the directional relation of parallelism and the velocity-ratio of equality.

To exemplify the translation of all the points of a moving piece in equal and similar curved paths, we may take the case of a coupling-rod (fig. 32) which connects together a pair of equal

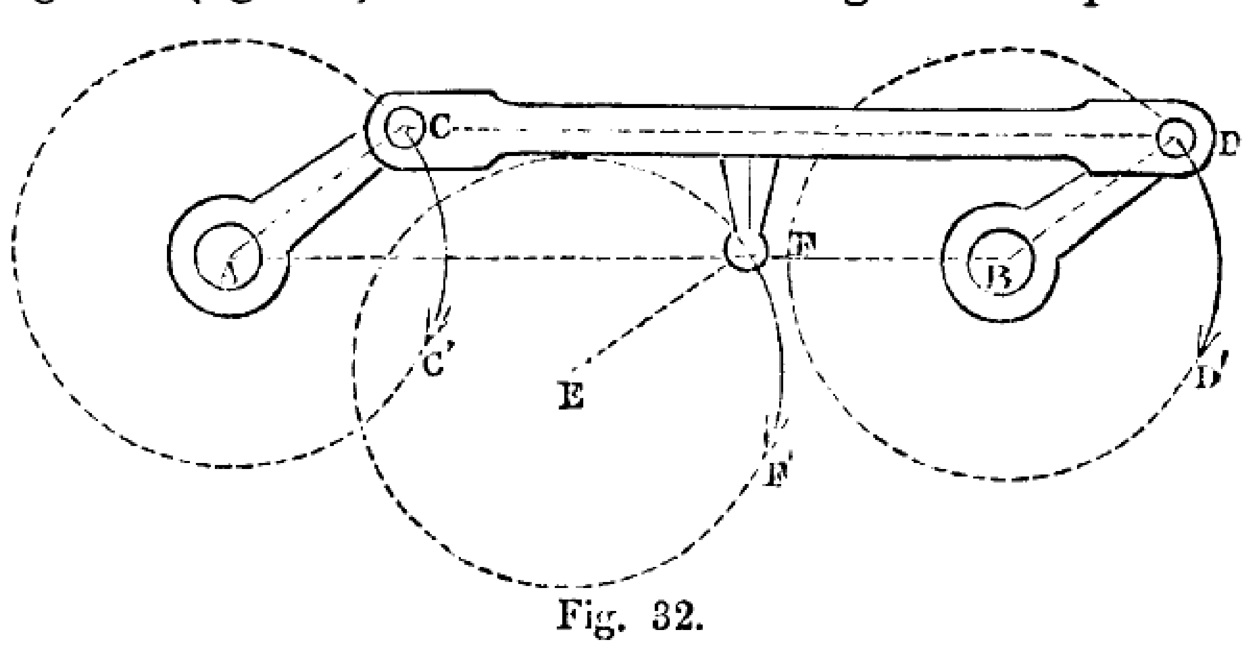

Fig. 32.

cranks, A C, B D, and has its effective length, C D, equal to the perpendicular distance, A B, between the axes of rotation of the two cranks. The motion of that coupling-rod is one of translation, in which all the particles describe with equal speed equal and similar circles of the radius A C = B D, in planes perpendicular to the axes A and B. The same is the case with any particle rigidly attached to the coupling-rod; such as F, which revolves in a circle of the radius E F = A C; so that, for example, the points C, D, and F move simultaneously through the equal and similar arcs C C′, D D′, F F′.

69. **Rotation Parallel to a Fixed Plane — Temporary Axis — Instantaneous Axis.**—The cases next in order as to complexity are those in which all the movements of the piece are parallel to a fixed plane; and the following are the problems which present themselves :—

I. *Given, the paths of two points in a moving piece, the distance between their projections on the plane of motion, and two successive positions of one of them, to find the* temporary axis of motion *of the piece.*

In fig. 33, let the plane of projection and of motion be that of

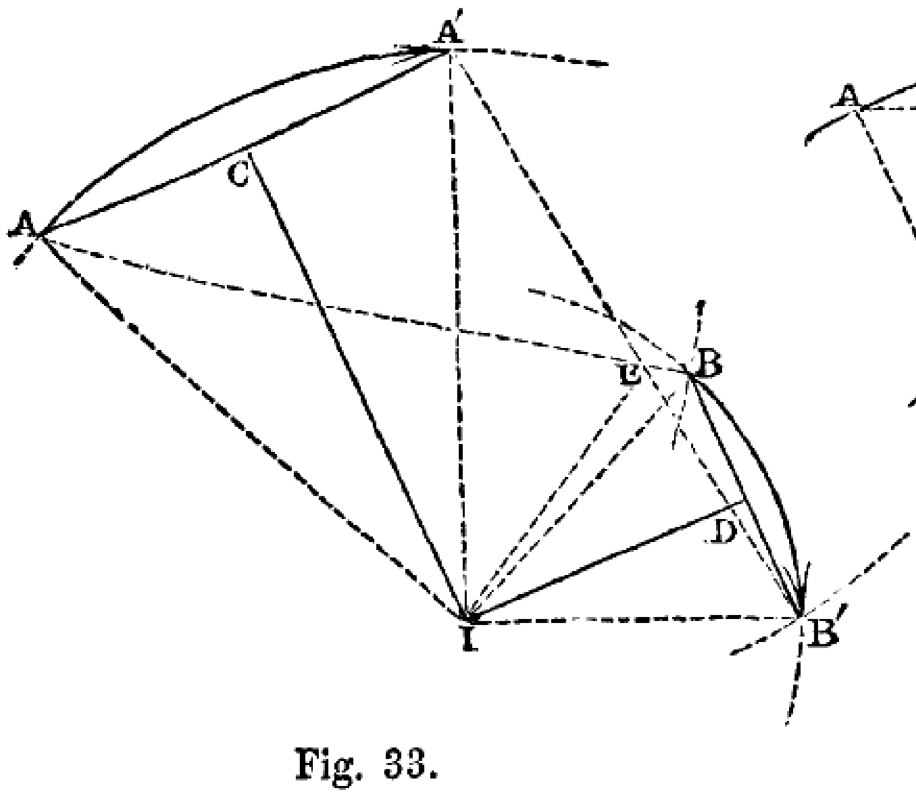

Fig. 33.

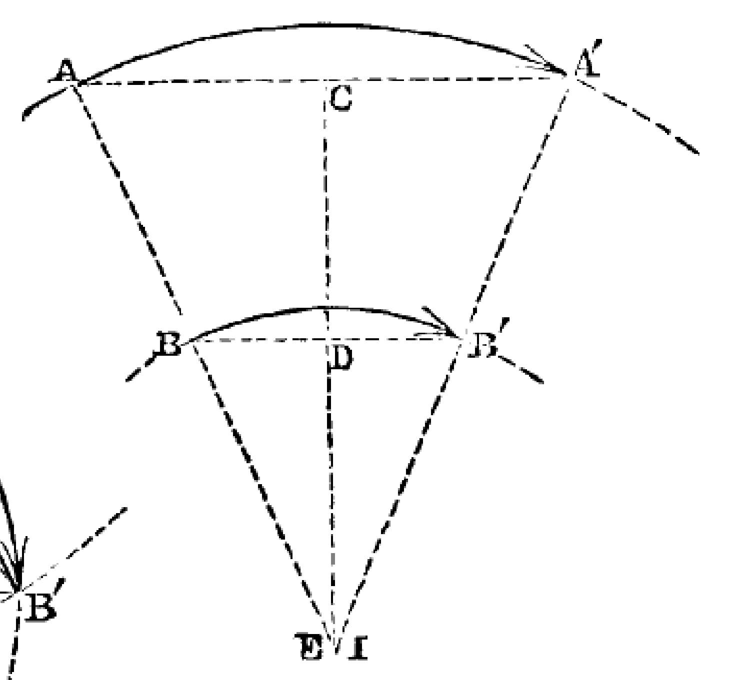

Fig. 34.

the paper, and let the partly dotted lines A A′ and B B′ be the projections of the paths of the two points, which may be straight lines or plane curves of any figure, subject only to the limitation that the distance between the points is invariable. Let A and A′ be the given two successive positions of one of the points. About A and A′ respectively, with the projection of the line of connection as a radius, draw circular arcs cutting the projected path of the other point in B and B′; these will be the projections of the two successive positions of the second point; and the straight lines A B and A′ B′ will be the projections of the line of connection in the two successive positions of the moving piece. Draw the straight lines A A′ and B B′; bisect them in C and D, through which points draw C I perpendicular to A A′ and D I perpendicular to B B′, meeting each other in I. Then, because A′ I = A I and B′ I = B I, I represents the same point in the two positions of the piece; and therefore I is the projection and the trace of a line perpendicular to the plane of motion, whose position is the same after the motion of A to A′ and of B to B′ that it was before. That line may be called the *Temporary Axis of Motion* of the moving piece, because the change of position of the piece is the same as if it had been turned through an angle A I A′ = B I B′ about that line.

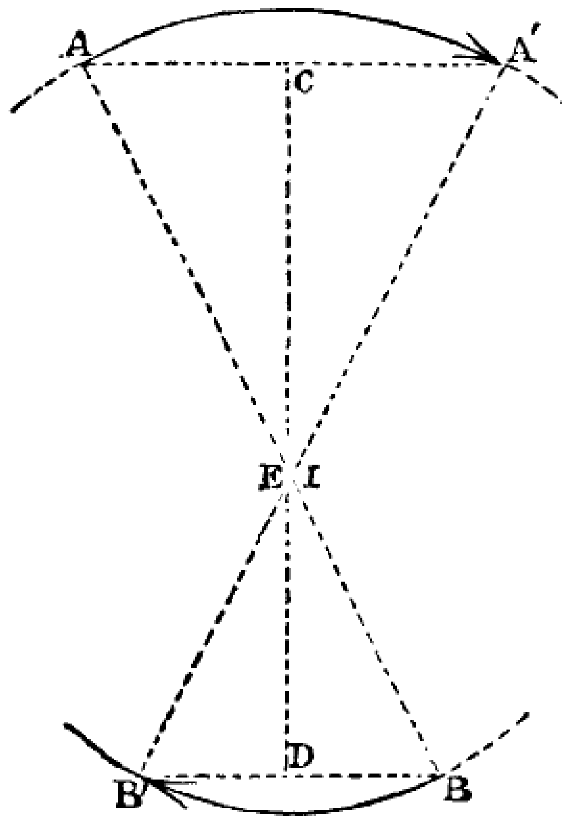

Fig. 35.

Let E be the point of intersection of A B and A′ B′. Then the straight line E I bisecting the angle A E B′ traverses the temporary axis I; and this affords a means of finding that axis when C I and D I cut each other at an angle so oblique as to make it difficult to determine precisely their point of intersection.

When B B′ is parallel to A A′, as in figs. 34 and 35, C I and D I become parts of one straight line, and have no intersection; and then the point I is determined by its coinciding with E. In most cases of this kind it is necessary that the two successive positions of B should be given as well as those of A.

II. *Given* (in fig. 36), *the projections* A *and* B, *at a given instant, of two points in a moving piece on the plane of motion, and the simultaneous directions of motion of those points,* A a *and* B b, *to find the* instantaneous axis *of the moving piece; and thence to deduce the comparative motions, at the given instant, of the given points, and of any other points in the moving piece.*

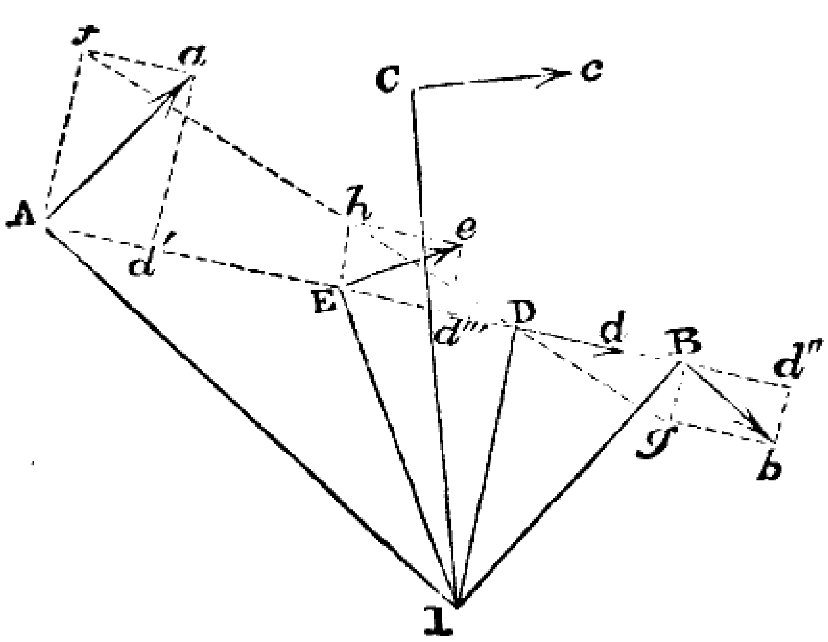
Fig. 36.

If the simultaneous directions of motion of the given points are perpendicular to their line of connection, the problem requires additional data for its solution, which will be stated in Rule III. If those directions are parallel to each other, and not perpendicular to the line of connection, the motion of the piece is one of translation, like that referred to in Article 68, page 44. The present rule comprehends all cases in which the given directions are not parallel to each other.

Through A and B draw A I and B I perpendicular respectively to A a and B b, and cutting each other in I. Then I will be the projection and the trace on the plane of motion of the required INSTANTANEOUS AXIS: that is to say, of a line such that the motion of the piece *at the instant in question* is one of rotation about that axis.

An instantaneous axis is so called because it is an imaginary line which is continually changing its position, both relatively to the frame of the machine and relatively to the secondary piece to which it belongs; so that it occupies any particular position, whether relatively to the frame or relatively to the secondary piece, at a particular instant only.

The comparative motions at the given instant of points in the secondary piece are deduced from the principle that the velocities of those points are proportional in magnitude and perpendicular in direction to the perpendiculars let fall from the points upon the instantaneous axis. For example, let A a, B b, C c, D d, E e, represent the directions and velocities of the points whose projections are A, B, C, D, E; then

$$\mathrm{A}a : \mathrm{B}b : \mathrm{C}c : \mathrm{D}d : \mathrm{E}e$$

are respectively proportional and perpendicular to

$$:: \mathrm{AI} : \mathrm{BI} : \mathrm{CI} : \mathrm{DI} : \mathrm{EI}.$$

From I let fall I D perpendicular to the projection, A B, of the line of connection of the given points. Then all points whose projections are at D are at the given instant in the act of moving parallel to A B; and all points whose projections are in A B, or in A B produced, such as A, B, and E, have for their component velocities along A B velocities equal to the velocity of D; that is to say,

$$\mathrm{D}d = \mathrm{A}d' = \mathrm{B}d'' = \mathrm{E}d''';$$

a consequence which follows also from the principle of Article 53, page 31.

The components perpendicular to A B of the velocities of points whose projections are in that line, such as A, B, and E, are proportional to the distances of those projections from D; that is to say, if A f, B g, and E h represent those transverse component velocities, we have the proportions,

$$\mathrm{DA} : \mathrm{DB} : \mathrm{DE}$$
$$:: \mathrm{A}f : \mathrm{B}g : \mathrm{E}h;$$

and the points f, h, D, g are in one straight line.