CHAPTER IV.

OF ELEMENTARY COMBINATIONS IN MECHANISM.

SECTION I.—*Definitions, General Principles, and Classification.*

89. **Elementary Combinations Defined.** (*A. M.*, 431.)—An "Elementary Combination" in Mechanism (a term introduced by Mr. Willis) consists of a pair of primary moving pieces, so connected that one transmits motion to the other. In other words, (to quote the Article *Mechanics* (*Applied*), in the eighth edition of the *Encyc. Brit.*)—

"An *elementary combination* in mechanism consists of two pieces whose kinds of motion are determined by their connection with the frame, and their comparative motion by their connection with each other; that connection being effected either by direct contact of the pieces, or by a connecting" (secondary) "piece" (such as a band, or a link, or a mass of fluid), "which is not connected with the frame, and whose motion depends entirely on the motions of the pieces which it connects."

"The piece whose motion is the cause is called the *driver;* the piece whose motion is the effect, the *follower.*"

"The connection of each of those two pieces with the frame is in general such as to determine the path of every moving point. In the investigation, therefore, of the comparative motion of the driver and follower, in an elementary combination, it is unnecessary to consider relations of angular direction, which are already fixed by the connection of each piece with the frame; so that the inquiry is confined to the determination of the velocity-ratio, and of the directional-relation so far only as it expresses the connection between *forward* and *backward* movements of the driver and follower. When a continuous motion of the driver produces a continuous motion of the follower, forward or backward, and a reciprocating motion a motion reciprocating at the same instant, the directional-relation is said to be *constant.* When a continuous motion produces a reciprocating motion, or *vice versâ;* or when a reciprocating motion produces a motion not reciprocating at the same instant, the directional-relation is said to be *variable.*"

90. **Line of Connection.**—In every class of elementary combinations, except those in which the connection is made by reduplication of cords, or by an intervening fluid, there is at least one straight

line called the *line of connection* of the driver and follower; being a line traversing a pair of points in the driver and follower respectively, which points are so connected that the component of their velocity relatively to each other, resolved along the line of connection, is null.

91. **Comparative Motions of Connected Points and Pieces.**—From the definition of a line of connection it follows, that *the components of the velocities of a pair of connected points along their line of connection are equal.* And from this, and from the property of a rigid body already stated in Article 54, page 32, it follows, that *the components, along a line of connection, of all the points traversed by that line, whether in the driver or in the follower, are equal.*

The general principle which has just been stated serves to solve every problem in which—the mode of connection of a pair of pieces being given—it is required to find their comparative motion at a given instant, or *vice versâ.*

The following are the rules for applying that principle to the three classes of problems which most frequently occur with reference to elementary combinations:—

I. *Pair of Points; or Pair of Sliding Pieces.*—In fig. 57, let A B be a line of connection; and let it be taken as the axis of projection. Let A be a point in the driver, and B a point in the follower, both in the line of connection. Let A a', A a'' be the two projections of the direction of motion of A at a given instant; and let B b', B b'' be the two projections of the direction of motion of B at the same instant. Lay off, along the line of connection and in the same direction, the equal distances A a = B b; draw $a''\,a\,a'$, $b''\,b\,b'$ perpendicular to the line of connection; then A a' and A a'', B b' and B b'' will be the projections of a pair of lines proportional respectively to the velocities of A and B at that instant. The lengths of those lines may be found by the Rule of Article 19, page 7.

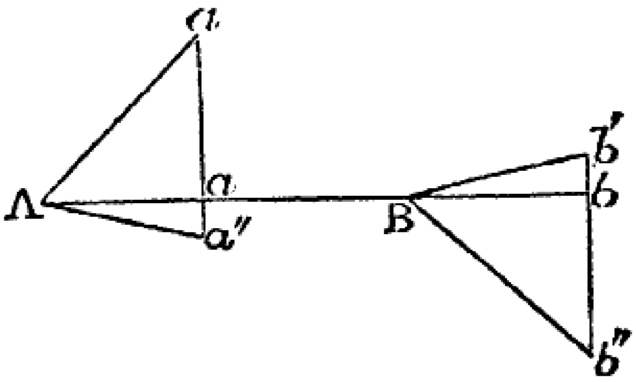

Fig. 57.

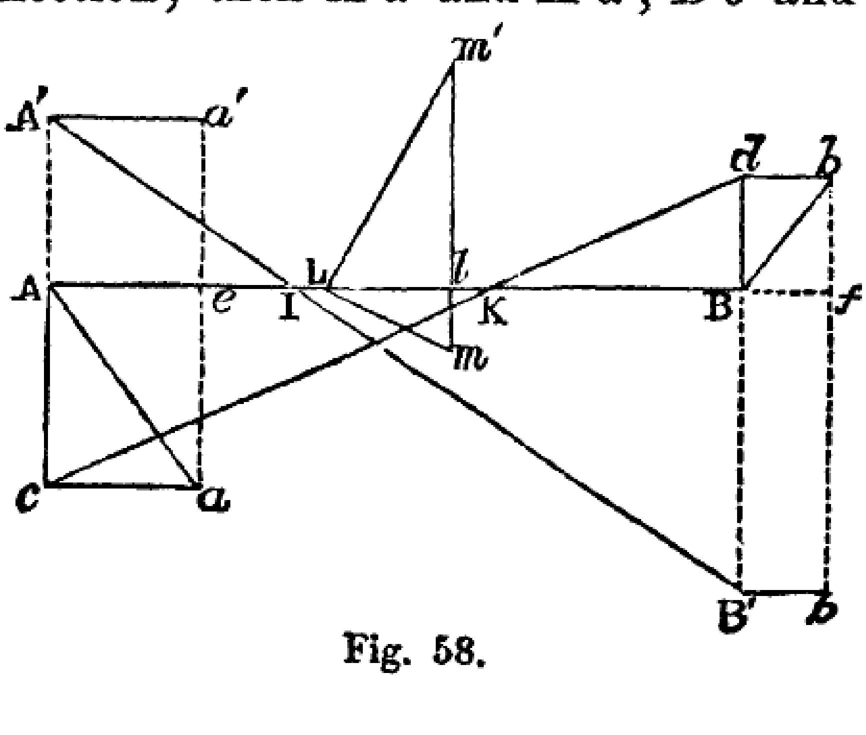

Fig. 58.

II. *Pair of Turning Pieces.*—In fig. 58, let A B be the line of connection of a pair of turning primary pieces. Let A and B be the points where that line is met by the common perpendiculars from the axes of rotation of the two pieces. (As to finding such common perpendiculars, see Article 36, page 15.) Let A A′ and B B′ be the *rabatments* of those two perpendiculars, drawn in opposite directions. Draw the straight line A′ B′ (called the *line of centres*), cutting the line of connection in I.

Then, because the component velocities of A and B along A B are equal, the angular velocities (or the component angular velocities) of the driver and follower about axes perpendicular to A B must be to each other *in the inverse ratio of the perpendiculars* A A′ *and* B B′; or, what is the same thing, *in the inverse ratio of the segments* I A′ *and* I B′ *into which the line of centres is cut by the line of connection.*

Hence the following construction:—In A B take A K = B I (or B K = A I); and through K draw an oblique straight line in any convenient direction, so as to cut A′ A produced in *c* and B′ B produced in *d*; then the component angular velocities of the pieces about two axes, A *c* and B *d*, perpendicular to the line of connection, will be to each other in the direct ratio of A *c* to B *d*. Also lay off, in opposite directions, the angles B A *a* and *f* B *b*, equal to the angles which the two axes of rotation respectively make with the line of connection, and draw *c a* and *d b* parallel to A B, cutting A *a* and B *b* in *a* and *b* respectively. Then *the ratio of* A *a* to B *b will be that of the resultant angular velocities of the two pieces.*

Through A′ and B′ draw A′ *a*′ and B′ *b*′ parallel to A B; and through *a* and *b* draw *a e a*′ and *b f b*′ perpendicular to A B. Then the proportion borne by *c a* = A *e* = A′ *a*′ to *d b* = B *f* = B′ *b*′ is that of the component angular velocities of the two pieces about axes parallel to the line of connection A B. Also A *a* and A′ *a*′ represent the projections of the axis of rotation of the first piece upon a pair of planes which cut each other in A *e*, one perpendicular and the other parallel to the common perpendicular whose rabatment is A A′; and B *b* and B′ *b*′ represent the projections of the axis of rotation of the second piece upon a pair of planes which cut each other in B *f*, one perpendicular and the other parallel to the common perpendicular whose rabatment is B B′

III. *Turning Piece and Sliding Piece.*—In fig. 58, let A L be the line of connection of a turning piece and a sliding piece, and let it be taken for the axis of projection; and let one of the planes of projection be parallel to the axis of the turning piece. Let A *a* and A *a*′ be the projections of that axis; so that A A′ perpendicular to A L is the common perpendicular of the axis and the line of connection. Take A *a* to represent the angular velocity of the turning piece, and from *a* draw *a c* parallel to L A, cutting A′ A (produced if necessary) in *c*. Then A *c* will represent the component angular velocity of the turning piece about an axis, A *c*, perpendicular to A L; and the product A A′· A *c* will represent *the component velocity of any point in* A L *along that line.*

Let L be a point in the line of connection and in the sliding piece; and let L *m* and L *m′* be the projections of the direction of motion of that piece. Lay off any convenient length, L *l*, to represent the component velocity of the sliding piece along the line of connection, and draw *m l m′* perpendicular to that line; then L *m* and L *m′* will represent *the two projections of the velocity of the sliding piece.*

Another construction is as follows:—Having determined the angle which the direction of motion of the sliding piece makes with the line of connection A L, draw A′ I, making the angle A A′ I equal to that angle; then the velocity of the sliding piece will be equal to that of *a point revolving at the end of the arm* A′ I, *with the angular velocity represented by* A *c.*

92. **Adjustments of Speed.**—The velocity-ratio of a driver and its follower is sometimes made capable of being changed at will, by means of apparatus for varying the position of their line of connection: as when a pair of rotating cones are embraced by a belt which can be shifted so as to connect portions of their surfaces of different diameters. Various such contrivances will be described in a later chapter.

93. A **Train of Mechanism** consists of a series of moving pieces, each of which is follower to that which drives it, and driver to that which follows it. In the case of a train of elementary combinations the comparative motion of the last follower and first driver is found by multiplying together all the velocity-ratios of the several elementary combinations of which the train consists, each ratio having the directional-relation with which it is connected denoted by means of the positive or negative algebraical sign, as the case may be. The product is the velocity-ratio of the last follower and first driver; and their directional-relation is indicated by the algebraical sign of that product, found by the rules, that any number of positive factors, and any even number of negative factors, give a positive product; and that any odd number of negative factors give a negative product.

94. **Elementary Combinations Classed.**—The only classification of elementary combinations that is founded, as it ought to be, on comparative motion, as expressed by velocity-ratio and directional-relation, is that first given by Mr. Willis in his Treatise on *Pure Mechanism.* Its general plan is as follows:—

Class A: Directional-relation constant; velocity-ratio constant.

Class B: Directional-relation constant; velocity-ratio varying.

Class C: Directional-relation changing periodically; velocity-ratio constant or varying.