# CHAPTER II.

## OF THE PERFORMANCE OF WORK BY MACHINES.

### SECTION I.—*Of Resistance and Work.*

297. The **Action of a Machine** is to produce Motion against Resistance. For example, if the machine is one for lifting solid bodies, such as a crane, or fluid bodies, such as a pump, its action is to produce upward motion of the lifted body against the resistance arising from gravity; that is, against its own weight: if the machine is one for propulsion, such as a locomotive engine, its action is to produce horizontal or inclined motion of a load against the resistance arising from friction, or from friction and gravity combined: if it is one for shaping materials, such as a planing machine, its action is to produce relative motion of the tool and of the piece of material shaped by it, against the resistance which that material offers to having part of its surface removed; and so of other machines.

298. **Work.** (*A. M.*, 513.)—The action of a machine is measured, or expressed as a definite quantity, by multiplying the motion which it produces into the resistance, or force directly opposed to that motion, which it overcomes; the product resulting from that multiplication being called WORK.

In Britain, the distances moved through by pieces of mechanism are usually expressed in feet; the resistances overcome, in pounds avoirdupois; and quantities of work, found by multiplying distances in feet by resistances in pounds, are said to consist of so many *foot-pounds.* Thus the work done in lifting a weight of one pound, through a height of one foot, is *one foot-pound;* the work done in lifting a weight of twenty pounds, through a height of one hundred feet, is $20 \times 100 = 2,000$ foot-pounds.

In France, distances are expressed in mètres, resistances overcome in kilogrammes, and quantities of work in what are called *kilogrammètres,* one kilogrammètre being the work performed in lifting a weight of one kilogramme through a height of one mètre.

The following are the proportions amongst those units of distance, resistance, and work, with their logarithms :—

| | | Logarithms. |
|---|---|---|
| One mètre | = 3·2808693 feet, | 0·515989 |
| One foot | = 0·30479721 mètres, | $\bar{1}$·484011 |
| One kilogramme | = 2·20462 lbs. avoirdupois, | 0·343334 |
| One lb. avoirdupois | = 0·453593 kilogramme, | $\bar{1}$·656666 |
| One kilogrammètre | = 7·23308 foot-pounds, | 0·859323 |
| One foot-pound | = 0·138254 kilogrammètres, | $\bar{1}$·140677 |

299. The **Rate of Work** of a machine means, the quantity of work which it performs in some given interval of time, such as a second, a minute, or an hour (*A. M.*, 661). It may be expressed in units of work (such as foot-pounds) per second, per minute, or per hour, as the case may be; but there is a peculiar unit of power appropriated to its expression, called a HORSE-POWER, which is, in Britain,

550 foot-pounds per second,
or 33,000 foot-pounds per minute,
or 1,980,000 foot-pounds per hour.

This is also called an *actual* or *real* horse-power, to distinguish it from a *nominal* horse-power, the meaning of which will afterwards be explained. It is greater than the performance of any ordinary horse, its name having a conventional value attached to it.

In France, the term FORCE DE CHEVAL, OR CHEVAL-VAPEUR, is applied to the following rate of work :—

| | | Foot-lbs. |
|---|---|---|
| 75 kilogrammètres per second | = | $542\frac{1}{2}$ |
| or 4,500 kilogrammètres per minute | = | 32,549 |
| or 270,000 kilogrammètres per hour | = | 1,952,932 |

being about one-seventieth part less than the British horse-power.

300. **Velocity.**—If the *velocity of the motion* which a machine causes to be performed against a given resistance be given, then the product of that velocity into the resistance obviously gives the rate of work, or effective power. If the velocity is given in feet per second, and the resistance in pounds, then their product is the rate of work in foot-pounds per second, and so of minutes, or hours, or other units of time.

It is usually most convenient, for purposes of calculation, to express the velocities of the parts of machines either in feet per second or in feet per minute. For certain dynamical calculations to be afterwards referred to, the second is the more convenient unit of time: in stating the performance of machines for practical purposes, the minute is the unit most commonly employed.

*Comparison of Different Measures of Velocity.*

| | Miles per hour. | Feet per second. | Feet per minute. | Feet per hour. |
|---|---|---|---|---|
| | 1 | = $1{\cdot}4\dot{6}$ | = 88 | = 5280· |
| | $0{\cdot}68\dot{1}\dot{8}$ | = 1 | = 60 | = 3600 |
| | $0{\cdot}011\dot{3}\dot{6}$ | = $0{\cdot}01\dot{6}$ | = 1 | = 60 |
| | $0{\cdot}00018\dot{9}\dot{3}$ | = $0{\cdot}0002\dot{7}$ | = $0{\cdot}01\dot{6}$ | = 1 |
| 1 nautical mile per hour, or "knot,"...... | = 1·1508 | = 1·688 | = 101·27 | = 6076 |

The units of time being the same in all civilized countries, the proportions amongst their units of velocity are the same with those amongst their linear measures.

301. **Work in Terms of Angular Motion.** (*A. M.*, 593.)—When a resisting force opposes the motion of a part of a machine which moves round a fixed axis, such as a wheel, an axle, or a crank, the product of the amount of that resistance into its *leverage* (that is, the perpendicular distance of the line along which it acts from the fixed axis) is called the *moment*, or *statical moment*, of the resistance. If the resistance is expressed in pounds, and its leverage in feet, then its moment is expressed in terms of a measure which may be called a *foot-pound*, but which, nevertheless, is a quantity of an entirely different kind from a foot-pound of work. (See p. 321.)

Suppose now that the body to whose motion the resistance is opposed turns through any number of revolutions, or parts of a revolution; and let T denote the angle through which it turns, expressed in revolutions, and parts of a revolution; also, let

$$2\,\pi = 6{\cdot}2832$$

denote, as is customary, the ratio of the circumference of a circle to its radius. Then the distance through which the given resistance is overcome is expressed by

$$\text{the leverage} \times 2\,\pi \times \mathrm{T};$$

that is, by the product of the circumference of a circle whose radius is the leverage, into the number of turns and fractions of a turn made by the rotating body.

The distance thus found being multiplied by the resistance overcome, gives the work performed; that is to say,

*The work performed*

$$= \textit{the resistance} \times \textit{the leverage} \times 2\,\pi \times \mathrm{T}.$$

But the product of the resistance into the leverage is what is called the *moment* of the resistance, and the product $2\pi T$ is called the *angular motion* of the rotating body; consequently,

*The work performed*
*= the moment of the resistance × the angular motion.*

The mode of computing the work indicated by this last equation is often more convenient than the direct mode already explained in Article 298.

The angular motion $2\pi T$ of a body during some definite unit of time, as a second or a minute, is called its *angular velocity;* that is to say, *angular velocity is the product of the turns and fractions of a turn made in an unit of time into the ratio* ($2\pi = 6{\cdot}2832$) *of the circumference of a circle to its radius.* Hence it appears that

*The rate of work*
*= the moment of the resistance × the angular velocity.*

302. **Work in Terms of Pressure and Volume.** (*A. M.*, 517.)—If the resistance overcome be a pressure uniformly distributed over an area, as when a piston drives a fluid before it, then the amount of that resistance is equal to the intensity of the pressure, expressed in units of force on each unit of area (for example, in pounds on the square inch, or pounds on the square foot) multiplied by the area of the surface at which the pressure acts, if that area is perpendicular to the direction of the motion; or, if not, then by the projection of that area on a plane perpendicular to the direction of motion. In practice, when the *area of a piston* is spoken of, it is always understood to mean the projection above mentioned.

Now, when a plane area is multiplied into the distance through which it moves in a direction perpendicular to itself, if its motion is straight, or into the distance through which its centre of gravity moves, if its motion is curved, the product is the *volume of the space traversed* by the piston.

Hence the work performed by a piston in driving a fluid before it, or by a fluid in driving a piston before it, may be expressed in either of the following ways:—

*Resistance × distance traversed*
*= intensity of pressure × area × distance traversed;*
*= intensity of pressure × volume traversed.*

In order to compute the work in foot-pounds, if the pressure is stated in pounds on the square foot, the area should be stated in square feet, and the volume in cubic feet; if the pressure is stated in pounds on the square inch, the area should be stated in square inches, and the volume in units, each of which is a prism of one foot in length and one square inch in area; that is, of $\frac{1}{144}$ of a cubic foot in volume.

The following table gives a comparison of various units in which the intensities of pressures are commonly expressed. (*A. M.*, 86.)

| | Pounds on the square foot. | Pounds on the square inch. |
|---|---|---|
| One pound on the square inch,..... | 144 | 1 |
| One pound on the square foot,..... | 1 | $\frac{1}{144}$ |
| One inch of mercury (that is, weight of a column of mercury at 32° Fahr., one inch high),........... | 70·73 | 0·4912 |
| One foot of water (at 39°·1 Fahr.), | 62·425 | 0·4335 |
| One inch of water,.................. | 5·2021 | 0·036125 |
| One atmosphere, of 29·922 inches of mercury, or 760 millimètres, | 2116·3 | 14·7 |
| One foot of air, at 32° Fahr., and under the pressure of one atmosphere,............................ | 0·080728 | 0·0005606 |
| One kilogramme on the square mètre, ............................ | 0·204813 | 0·00142231 |
| One kilogramme on the square millimètre, ........................ | 204813 | 1422·31 |
| One millimètre of mercury,......... | 2·7847 | 0·01934 |

303. **Algebraical Expressions for Work.** (*A. M.*, 515, 517, 593.)—To express the results of the preceding articles in algebraical symbols, let

$s$ denote the distance in feet through which a resistance is overcome in a given time;

R, the amount of the resistance overcome in pounds.

Also, supposing the resistance to be overcome by a piece which turns about an axis, let

T be the number of turns and fractions of a turn made in the given time, and $i = 2\pi T = 6·2832\ T$ the angular motion in the given time; and let

$l$ be the leverage of the resistance; that is, the perpendicular distance of the line along which it acts from the axis of motion; so that $s = i\,l$, and R $l$ is the statical moment of the resistance. Supposing the resistance to be a pressure, exerted between a piston and a fluid, let A be the area or projected area of the piston, and $p$ the intensity of the pressure in pounds per unit of area.