CHAPTER IV.

OF THE EFFICIENCY AND COUNTER-EFFICIENCY OF PIECES, COMBINATIONS, AND TRAINS, IN MECHANISM.

370. **Nature and Division of the Subject.**—The terms *Efficiency* and *Counter-efficiency* have already been explained in Article 335, page 377; and the laws of friction, the most important of the wasteful resistances which cause the efficiency of a machine to be less than unity, have been stated in Articles 309 to 311, pages 348 to 355. In the present Chapter are to be set forth the effects of wasteful resistance, and especially of friction, on the efficiency and counter-efficiency of single pieces, and of combinations and trains of pieces, in Mechanism. In practical calculations the *counter-efficiency* is in general the quantity best adapted for use; because the useful work to be done in an unit of time, or *effective power*, is in general given; and from that quantity, by multiplying it by the counter-efficiency of the machine—that is, by the continued product of the counter-efficiencies of all the successive pieces and combinations by means of which motion is communicated from the driving-point to the useful working-point—is to be deduced the value of the expenditure of energy in an unit of time, or *total power*, required to drive the machine. In symbols, let U be the useful work to be done per second; c, c', c'', &c., the counter-efficiencies of the several parts of the train; T, the total energy to be expended per second; then

$$\mathrm{T} = c \cdot c' \cdot c'' \cdot \&c.\ldots \mathrm{U}. \quad \ldots\ldots\ldots\ldots\ldots\ldots(1.)$$

When the mean effort required at the driving-point can conveniently be computed by reducing each resistance to the driving-point, and adding together the reduced resistances (as in Article 324, page 369, and Article 338, page 379), the ratio in which the actual effort required at the driving-point is greater than what the required effort would be, in the absence of wasteful resistance, is expressed by the continued product of the counter-efficiencies of the parts of the train, as follows: let P_0 be the effort required, in the absence of wasteful resistance; P, the actual effort required; then

$$\mathrm{P} = c \cdot c' \cdot c'' \cdot \&c.\ldots \mathrm{P}_0; \quad \ldots\ldots\ldots\ldots\ldots\ldots(2.)$$

and in determining the efficiency or the counter-efficiency of a single piece, the most convenient method of proceeding often consists in comparing together the efforts required to drive that piece, with and without friction, and thus finding the ratios

$$\frac{P_0}{P} = \text{efficiency}; \quad \frac{P}{P_0} = \text{counter-efficiency.} \quad \ldots\ldots\ldots(3.)$$

In the ensuing sections of this Chapter, the efficiency of single primary pieces is first treated of, and then that of the various modes of connection employed in elementary combinations.

Section I.—*Efficiency and Counter-efficiency of Primary Pieces.*

371. **Efficiency of Primary Pieces in General.**—A primary piece in mechanism, moving with an uniform velocity, is balanced under the action of four forces, viz. :—

I. The re-action of the piece which it drives: this may be called the *Useful Resistance*, and denoted by R;

II. The *weight* of the piece itself: this may be denoted by W.

III. The *effort* by which the piece is driven: this may be denoted by P; and its values with and without friction by P and P_0 respectively.

IV. The resultant pressure at the bearings, or *bearing-pressure*, which may be denoted by Q; and which of course is equal and directly opposed to the resultant of the first three forces.

In the absence of friction, the bearing-pressure would be normal to the bearing-surface. The effect of friction is, that the line of action of the bearing-pressure becomes oblique to the bearing-surface, making with the normal to that surface the angle of repose (ϕ), whose tangent ($f = \tan \phi$) is the co-efficient of friction (see Article 309, page 349); and the amount of the friction is expressed by $Q \sin \phi$, or very nearly by fQ, when the co-efficient of friction is small. (3)

In the class of problems to which this Chapter relates, the first two forces—that is, the useful resistance R, and the weight W—are given in magnitude, position, and direction; and in most cases it is convenient to find their resultant, in magnitude, position, and direction, by the rules of statics: that is to say, if the line of action of R is vertical, by Rule I. of Article 280, page 322; and if inclined, by the Rules given or referred to in Article 278, page 319. In what follows, the resultant of the useful resistance and weight will be called the *given force*, and denoted by R′.

The third force—that is, the effort required in order to drive the piece at an uniform speed—is given in position and direction; for its line of action is the line of connection of the piece under consideration with the piece that drives it. The magnitude of the effort is one of the quantities to be found.

The fourth force—that is, the bearing-pressure—has to be found in position, direction, and magnitude. The general principles according to which it is determined are the following:—

First, That if the lines of action of the given force and the effort are parallel to each other, the line of action of the resultant bearing-pressure must be parallel to them both; and that if they are inclined to each other, the line of action of the resultant bearing-pressure must traverse their point of intersection.

Secondly, That at the centre of pressure, where the line of action of the resultant bearing-pressure cuts the bearing-surfaces, it makes an angle with the common normal of those surfaces equal to their angle of repose, and in such a direction that its tangential component (being the friction) is directly opposed to the relative sliding motion of that pair of surfaces over each other.

Thirdly, That the given force, the effort, and the bearing-pressure, form a system of three forces that balance each other; and are therefore proportional to the three sides of a triangle parallel respectively to their directions.

371 A. **Conditions Assumed to be Fulfilled.**—In all the problems treated of in this section, the following conditions are assumed to be fulfilled:—*First*, that except when otherwise specified, the forces other than bearing-pressures which are applied to the piece under consideration—that is, the useful resistance, the weight, and the effort—act either in parallel directions, or exactly or nearly in one plane, parallel to the planes of motion of the particles of the piece; *secondly*, that the acting parts of the piece do not *overhang* the bearings; and *thirdly*, that the bearing-surfaces fit each other easily without any grasping or pinching. As to the object of the fulfilment of such conditions, and the effects of departure from them, the following explanations have to be made:—

I. The bearing-surface of many primary pieces, and especially of rotating pieces, is in general divided into two parts; for example, an axle is very often supported by two journals. If the forces other than bearing-pressures which are applied to the moving piece, are parallel to each other, the parts of the bearing-pressure will also be parallel to them and to each other; and the sum of the frictional resistances due to the two parts of the bearing-pressure will be simply equal to the frictional resistance due to the whole bearing-pressure treated as one force. The same will be the case when the forces other than bearing-pressures act in one plane, parallel to the planes of motion of the particles of the piece; and will be nearly the case when, although those forces act in different planes, the transverse distance between their planes of action is small compared with the distance between the planes of action of the two components into which the bearing-pressure is divided.

But when that condition is not fulfilled, the friction at the bearings, being proportional to the sum of the two components into which the bearing-pressure is divided, will be greater than the friction due simply to the resultant bearing-pressure considered as one force; and the efficiency of the piece will be diminished.

II. The effect upon the friction, and upon the work lost in overcoming it, produced when the acting parts of a moving piece *overhang* its bearings, may be approximately calculated and allowed for in the following manner:—

Suppose that the bearing-surface of a primary piece, whether sliding or turning, is divided into two parts; and that the transverse distance between the centres of those two parts—that is, the distance in a direction perpendicular to the planes of motion of the particles of the piece—is denoted by c. Let the plane of action of the forces other than bearing-pressures be situated *outside* the space between the two parts of the bearing-surface, and at the transverse distance z from the centre of the nearer of those parts; and consequently at the distance $z + c$ from the centre of the further of them. Let Q be the resultant bearing-pressure. The two components of that resultant pressure, exerted at the two parts of the bearing-surface, will be contrary to each other in direction; and their values will be respectively, (?)

at the nearer part, $\dfrac{Q(z + c)}{c}$;

and at the further part, $\dfrac{-Qz}{c}$.

The total friction will be the sum of two components exerted at the two parts of the bearing-surface respectively, and will be proportional to the *arithmetical sum* of the two components of the bearing-pressure; that is, to the force

$$\frac{Q(c + 2z)}{c};$$

whereas, if the plane of action of the resultant of the given force and the effort had not overhung the bearings, the friction would have been simply proportional to Q. Hence the effect of that plane's overhanging the bearings by the distance z, is to increase the friction approximately in the ratio of

$$1 + \frac{2z}{c} : 1.$$

III. As to the condition that the bearing-surfaces should fit each other easily, it is necessary in order that the bearing-pressure may not contain, to any appreciable extent, pairs of components which balance each other, being transverse to the direction of the resultant bearing-pressure; for such components cause an unnecessary addition to the friction. The ratio in which the friction of a *tight-fitting bearing* exceeds that of an easy-fitting bearing of the same dimensions and figure, is very nearly equal to that in which the whole area of the bearing-surface exceeds the area of the projection of that surface on a plane normal to the direction of the resultant bearing-pressure.

When the use of bearing-surfaces in pairs, oblique to the plane of the pressure and motion, is unavoidable (as, for example, in the case of the V-shaped bearings of a planing machine), their effect may be allowed for by increasing the co-efficient of friction in the ratio above-mentioned; which is expressed by the secant of the equal angles which the normals to the bearing-surfaces make with that plane.

372. **Efficiency of a Straight-sliding Piece.**—In fig. 263, let A A be a straight guiding-surface, upon which there slides, in the direc-

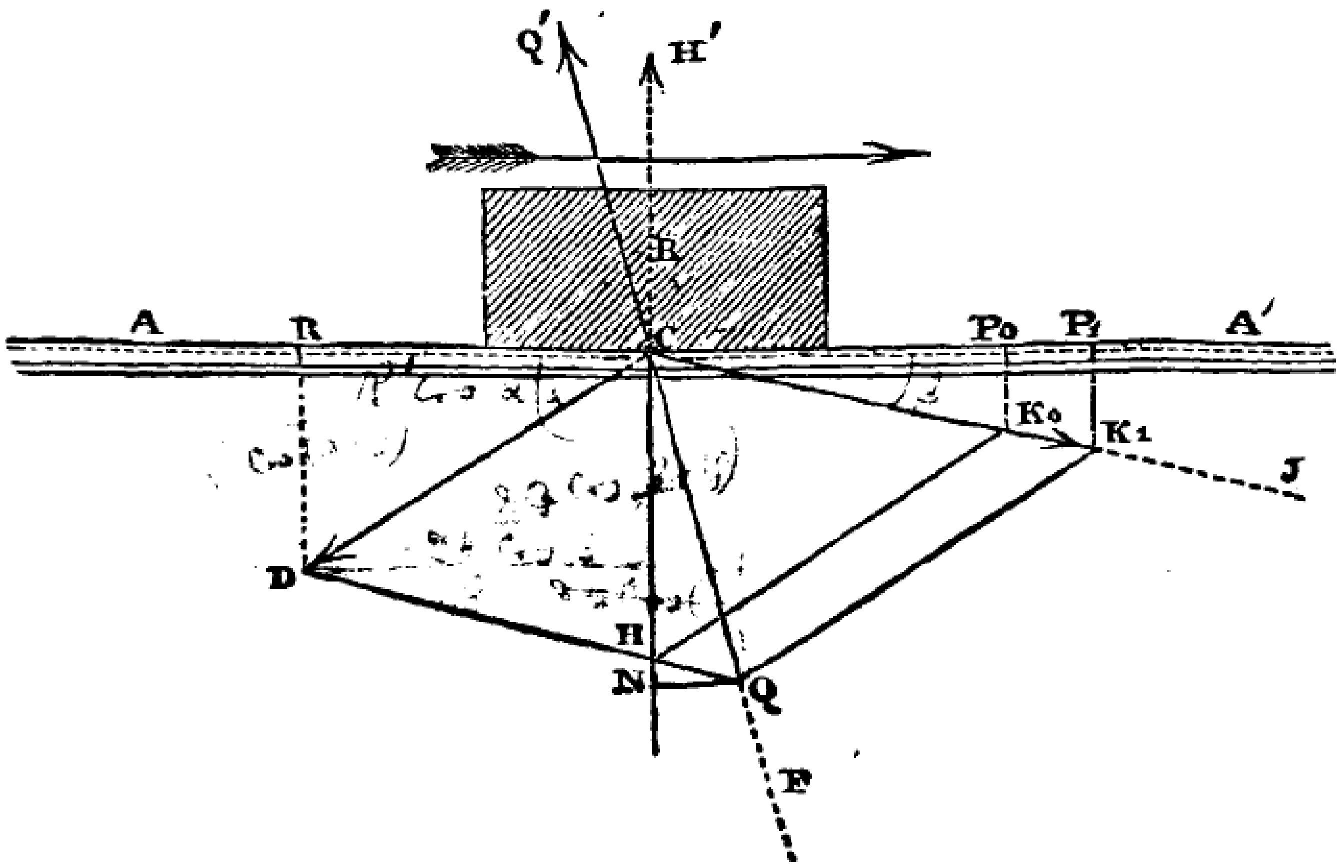

Fig. 263.

tion marked by the feathered arrow, the moving piece B. Let C D represent the *given force*, being the resultant of the useful resistance and of the weight of the piece B. (The figure shows the motion of B as horizontal; but it may be in any direction.) Let C J be the line of action of the effort by which the piece B is driven.

Draw C N perpendicular to A A; and C F making the angle N C F = the angle of repose. Through D, parallel to C J, draw the straight line D H Q, cutting C N in H, and C F in Q; and through H and Q, and parallel to D C, draw H K_0 and Q K_1, cutting C J in K_0 and K_1 respectively. Produce H C to H′, and Q C to Q′, making C H′ = H C, and C Q′ = Q C.