CHAPTER II.

PRINCIPLES AND RULES RELATING TO STRENGTH AND STIFFNESS.

411. The **Object of this Chapter** is to give a summary of the principles, and of the general rules of calculation, which are applicable to problems of strength and stiffness, whatsoever the particular material may be. It is to a certain extent identical with a similar summary which appeared in *A Manual of Civil Engineering*, but modified to adapt its principles to the problems which occur in machinery. Various special problems relating to machinery will be considered in the third Chapter.

SECTION I.—*Of Strength and Stiffness in General.*

412. **Load, Stress, Strain, Strength.**—The *load*, or combination of external forces, which is applied to any piece, moving or fixed, in a machine, produces *stress* amongst the particles of that piece, being the combination of forces which they exert in resisting the tendency of the load to disfigure and break the piece, accompanied by *strain*, or alteration of the volumes and figures of the whole piece, and of each of its particles.

If the load is continually increased, it at length produces either *fracture* or (if the material is very tough and ductile) such a disfigurement as is practically equivalent to fracture, by rendering the piece useless.

The *Ultimate Strength* of a body is the load required to produce fracture in some specified way. The *Proof Strength* is the load required to produce the greatest strain of a specific kind consistent with safety; that is, with the retention of the strength of the material unimpaired. A load exceeding the proof strength of the body, although it may not produce instant fracture, produces fracture eventually by long-continued application and frequent repetition.

The *Working Load* on each piece of a machine is made less than the ultimate strength, and less than the proof strength, in certain ratios determined partly by experiment and partly by practical experience, in order to provide for unforeseen contingencies.

Each solid has as many different kinds of strength as there are different ways in which it can be strained or broken, as shown in the following classification:—

	Strain.	Fracture.
Elementary......	Extension	......Tearing.
	Compression	......Crushing.
Compound.......	Distortion	......Shearing.
	Twisting	......Wrenching.
	Bending	......Breaking across.

413. **Co-efficients or Moduli of Strength** are quantities expressing the *intensity* of the stress under which a piece of a given material gives way when strained in a given manner; such intensity being expressed in units of weight for each unit of sectional area of the layer of particles at which the body first begins to yield. In Britain, the ordinary unit of intensity employed in expressing the strength of materials is the *pound avoirdupois on the square inch.* As to other units, see Article 302, page 342.

Co-efficients of strength are of as many different kinds as there are different ways of breaking a body. Their use will be explained in the sequel. Tables of their values are given at the end of the volume.

Co-efficients of strength, when of the same kind, may still vary according to the direction in which the stress is applied to the body. Thus the tenacity, or resistance to tearing, of most kinds of wood is much greater against tension exerted along than across the grain.

414. **Factors of Safety.**—A factor of safety, in the ordinary sense, is the ratio in which the load that is just sufficient to overcome instantly the strength of a piece of material is greater than the greatest safe ordinary working load.

The proper value for the factor of safety depends on the nature of the material; it also depends upon how the load is applied. The load upon any piece in a structure or in a machine is distinguished into *dead load* and *live load.* A *dead load* is a load which is put on by imperceptible degrees, and which remains steady; such as the weight of a structure, or of the fixed framing in a machine. A *live load* is one that is or may be put on suddenly, or accompanied with vibration; like a swift train travelling over a railway bridge; or like most of the forces exerted by and upon the moving pieces in a machine.

It can be shown that in most cases which occur in practice a live load produces, or is liable to produce, *twice,* or very nearly twice, the effect, in the shape of stress and strain, which an equal dead load would produce. The *mean* intensity of the stress produced by a suddenly applied load is no greater than that produced by the same load acting steadily; but in the case of the suddenly applied load, the stress begins by being insensible, increases to double its mean intensity, and then goes through a series of fluctuations, alternately below and above the mean, accompanied by vibration of the strained body. Hence the ordinary practice is to make the factor of safety for a live load *double* of the factor of safety for a dead load.

A distinction is to be drawn between *real* and *apparent* factors of safety. A real factor of safety is the ratio in which the ultimate or breaking stress is greater than the real working stress at the time when the straining action of the load is greatest. The apparent factor of safety has to be made greater than the real factor of safety in those cases in which the calculation of strength is based, not upon the greatest straining action of the load, but upon a mean straining action, which is exceeded by the greatest straining action in a certain proportion. In such cases the apparent factor of safety is the product obtained by multiplying the real factor of safety by the ratio in which the greatest straining action exceeds the mean.

Another class of cases in which the apparent exceeds the real factor of safety is when there are additional straining actions besides that due to the transmission of motive power, and when those additional actions, instead of being taken into account in detail, are allowed for in a rough way by means of an increase of the factor of safety. A third class of cases is when there is a possibility of an increased load coming by accident to act upon the piece under consideration. For example, a steam engine may drive two lines of shafting, exerting half its power on each; one may suddenly break down, or be thrown out of gear, and the engine may for a short time exert its whole power on the other.

The following table shows the ordinary values of real factors of safety:—

	Real Factors of Safety. Dead Load.	Real Factors of Safety. Live Load.
Perfect materials and workmanship, ...	2	4
Ordinary materials and workmanship—		
Metals, ...	3	6
Wood, Hempen Ropes, ...from	3 to 5	10
Masonry and Brickwork, ...	4	8

The following are examples of apparent factors of safety:—

Real Factor of Safety, 6.	Ratio in which Greatest Effort exceeds Mean Effort, nearly.	Apparent Factor of Safety.
Steam engines acting against a constant resistance—		
Single engine, ...	1·6	9·6
Pair of engines driving cranks at right angles, ...	1·1	6·6
Three engines driving equiangular cranks, ...	1·05	6·3

Ordinary cases of varying effort and resistance, ...	2·0	12·0
Lines of shafting in millwork; apparent factor of safety for twisting stress due to motive power, to cover allowances for bending actions, accidental extra load, &c., ...	from 18 to 36	

Almost all the experiments hitherto made on the strength of materials give co-efficients or moduli of *ultimate strength;* that is, co-efficients expressing the intensity of the stress exerted by the most severely strained particles of the material just before it gives way. In calculations for the purpose of designing framework or machinery to bear a given working load, there are two ways of using the factor of safety,—one is, to multiply the working load by the factor of safety, so as to determine the breaking load, and use this load in the calculation, along with the modulus of ultimate strength: the other is, to divide the modulus of ultimate strength by the factor of safety, and thus to find a modulus or co-efficient of *working stress,* which is to be used in the calculation, along with the *working load.* It is obvious that the two methods are mathematically equivalent, and must lead to the same result; but the latter is on the whole the more convenient in designing machines.

415. The **Proof** or **Testing** by experiment of the strength of a piece of material is conducted in two different ways, according to the object in view.

I. If the piece is to be *afterwards used,* the testing load must be so limited that there shall be no possibility of its impairing the strength of the piece; that is, it must not exceed the *proof strength*, being from one-third to one-half of the ultimate strength. About double or treble of the working load is in general sufficient. Care should be taken to avoid vibrations and shocks when the testing load approaches near to the proof strength.

II. If the piece is to be *sacrificed* for the sake of ascertaining the strength of the material, the load is to be increased by degrees until the piece breaks, care being taken, especially when the breaking point is approached, to increase the load by small quantities at a time, so as to get a sufficiently precise result.

The *proof strength* requires much more time and trouble for its determination than the ultimate strength. One mode of approximating to the proof strength of a piece is to apply a moderate load and remove it, apply the same load again and remove it, two or three times in succession, observing at each time of application of the load the *strain* or alteration of figure of the piece when loaded, by stretching, compression, bending, distortion, or twisting, as the