

## CHAPTER III.

OF SPECIAL PRINCIPLES RELATING TO STRENGTH AND STIFFNESS  
IN MACHINES.

**445. Subjects of this Chapter.**—In the designing of machines with a view to sufficient strength and stiffness, certain special principles must be kept in view besides those general principles which are applicable to machines in common with structures. The first section of this Chapter gives a summary of those principles; the remaining sections relate to the strength and stiffness of certain special parts of machines.

SECTION I.—*Summary of Principles.*

**446. Load in Machines.**—In most examples of machinery the whole load must be treated as a live load, because of its action being accompanied with vibration; and also in many cases because the straining action of the load operates upon different sets of particles in succession, and comes with more or less suddenness upon such sets of particles. In some of these latter cases the straining action of the load upon a given particle is periodically reversed; for example, the bending moment exerted on a rotating shaft causes alternate tension and thrust to be exerted upon the same particle, as it passes alternately to the stretched and to the compressed side of the axle.

Hence the real factor of safety in machinery is seldom less than 6.

There are exceptional cases in which, owing to the smoothness of the motion and the steadiness of the straining action, the load may be considered as intermediate between a dead load and a live load, so that a smaller factor of safety is sufficient; such, for example, as the transmission of power through bands of such length as to hang in a sensibly curved form.

**447. Straining Actions computed from Power.**—The straining actions on moving pieces can be in some cases wholly, and in others partly, determined from the power transmitted, and from the speed, by methods of calculation which will be described and exemplified further on. The cases in which the straining action can be wholly determined from the power transmitted are those which fulfil the following conditions: uniformity of effort, absence of *lateral components* in the straining forces, and smallness of the straining.

actions due to the weight and to the re-action of the piece itself, and of pieces carried by it, so that those parts of the straining action may be treated as insensible.

The rules for computing straining actions from power transmitted are the following:—

I. To compute the effort exerted along a given line of connection; divide the power transmitted, in units of work per second, by the common component along the line of connection of the velocities of the connected points.

If the power is given in horses-power, reduce it in the first place to units of work per second, by multiplying by 550 for foot-lbs., or by 75 for kilogrammètres.

II. To compute the straining moment exerted through a given rotating piece; divide the power transmitted, in units of work in a given time, by the angular motion in the same time: that is, by  $2\pi$  times the number of turns in that time.

In symbols, let  $U$  be the power, in units of work per minute;  $N$ , the number of revolutions per minute;  $M$ , the straining moment; then

$$M = \frac{U}{2\pi N} = \frac{0.159155 U}{N} \dots\dots\dots(1.)$$

This formula gives the moment in the same denomination with the work. If the work is given in foot-lbs. per minute, and the moment is required in inch-lbs., the above expression must be multiplied by 12; that is,

$$M = \frac{12 U}{2\pi N} = \frac{1.91 U}{N} \dots\dots\dots(2.)$$

Let  $H P$  denote the number of horses-power transmitted, so that

$$U \text{ in foot-lbs. per minute} = 33000 H P; \text{ and} \\ U \text{ in kilogrammètres per minute} = 4500 H P;$$

then we have

$$M \text{ in inch-lbs.} = \frac{63000 H P}{N}; \dots\dots\dots(3.)$$

$$M \text{ in foot-lbs.} = \frac{5250 H P}{N}; \dots\dots\dots(4.)$$

$$M \text{ in kilogrammètres} = \frac{716.2 H P}{N} \dots\dots\dots(5.)$$

The formula for kilogrammètres is adapted to the French horse-power, which is about one-seventieth part less than the British.

In the cases in which part only of the straining action can be determined from the power transmitted, the causes of additional straining action are the following:—Excess of maximum effort above mean effort; lateral components in straining forces; weight of the piece itself and of pieces carried by it; re-actions of the piece itself and of pieces carried by it, when undergoing acceleration or retardation. It has already been stated in Article 414, page 488, that such additional straining actions are sometimes calculated expressly, and sometimes allowed for by using an apparent factor of safety greater than the mean factor of safety in a suitable proportion.

There are cases in which the best method of calculating the straining action is to determine directly the greatest load, without reference to the power transmitted.

**448. Alternate Strains.**—Pieces are often met with in machinery which are strained alternately in opposite directions, such being especially the case when the motion is reciprocating: for example, the piston-rod and connecting-rod of a steam engine, which are subjected alternately to tension and to thrust; and the beam of a steam engine, which is exposed alternately to bending actions in opposite directions. Such pieces must be adapted to resist efficiently the straining action in either direction, and especially that which is most severe. This principle is applicable to framing as well as to moving pieces.

**449. Straining Effects of Re-action.**—When the particles of a piece undergo changes of speed and direction, their re-actions produce straining effects resembling those produced by their weights; due regard being had to the directions of those re-actions, and to the ratios which they bear to the weights of the particles. For example, if a particle of the weight  $w$  undergoes the acceleration  $d v$ , in the time  $d t$ , the re-action of that particle is

$$\frac{w d v}{g d t}, \dots \dots \dots (1.)$$

and is exerted in a direction opposite to that of the acceleration (Article 287, page 330); and if a particle of the weight  $w$  revolves with the angular velocity  $\alpha$ , in a circle of the radius  $r$ , its re-action (or centrifugal force) is

$$\frac{w \alpha^2 r}{g}, \dots \dots \dots (2.)$$

and is exerted in a direction away from the centre of the circle (Article 288, page 330).

In many cases of reciprocating motion in machinery, the motion of the reciprocating mass is *harmonic* (as to the meaning of which, see Article 239, page 250); and then its greatest re-action is equal

to what its centrifugal force would be if it revolved in a period equal to the time of a double stroke, in a circle of a radius equal to the half-stroke. Let  $T$  be the period, or time of a double stroke in seconds;  $x$ , the half-stroke;  $w$ , the weight of the reciprocating mass; then its greatest re-action is

$$w \frac{4 \pi^2}{g} \cdot \frac{w x}{T^2} \dots \dots \dots (3.)$$

The co-efficient  $\frac{4 \pi^2}{g}$  is the reciprocal of  $\frac{g}{4 \pi^2}$ , which is, as already stated in Article 319, page 364, the altitude of a revolving pendulum whose period is one second; that is, nearly, 0.815 foot, or 9.78 inches, or 248 millimètres.

The *moment of re-action* of a mass which undergoes an acceleration of angular velocity,  $d a$ , in the interval of time  $d t$ , is given by the expression

$$- \frac{I d a}{g d t}; \dots \dots \dots (4.)$$

in which  $I$  denotes the *moment of inertia* of the rotating mass (Article 313, page 358). If the mass has a *rocking* or oscillating motion, following the harmonic law, about its axis, the greatest moment of re-action is as follows:—

$$\frac{4 \pi^2}{g} \cdot \frac{I \theta}{T^2}; \dots \dots \dots (5.)$$

in which  $T$  is the periodic time of a complete or double oscillation, and  $\theta$  the *semi-amplitude*; that is, the angle in circular measure through which the stroke, or oscillation, extends to each side of the middle position of the rocking body. Values of  $\frac{g}{4 \pi^2}$  have already been given.

The moments of re-action given by the formulæ (4) and (5) may constitute twisting moments upon shafts, or bending moments upon levers.

450. **Framework.**—The load which strains the framework of a machine consists partly of the weight of that framework itself; but principally of the bearing-pressures exerted by the moving pieces. How those bearing-pressures are to be determined has already been shown in the course of Part II., Chapter IV., Section I. The framework ought to be so designed as to make the bearing-pressures at different points, or the components of those bearing-pressures, as far as possible balance each other. When this principle is perfectly carried out, the pressure exerted by the machine on its foundation will consist simply of its weight; all

